Mistuning and Friction Damping Interplay in Bladed Disks: Modeling Approaches and Design Perspectives

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***Abstract*—Real bladed disks are never perfectly identical from blade to blade: manufacturing tolerance, material inhomogeneity, and in-service wear introduce small blade-to-blade variations — mistuning — that can localize vibratory energy onto a small subset of blades and amplify their response substantially beyond the nominal, tuned prediction. Friction damping, the subject of the authors' companion reviews in this series, is the primary countermeasure available to a designer, yet mistuning and friction interact nontrivially rather than acting as independent, separable effects. This review surveys that interaction: the fundamentals of mistuning and the reduced-order modelling techniques developed to predict it efficiently, the smaller but growing literature that couples these techniques with nonlinear friction contact models and the compounding computational cost that coupling introduces, the particular challenge posed by integrally bladed disks (blisks) that cannot accommodate conventional under-platform dampers, the experimental evidence available to validate combined mistuning-friction predictions, and the uncertainty-quantification methods developed to bound worst-case response when both mistuning and friction-contact parameters are treated as uncertain. The review closes by connecting this literature explicitly to the authors' companion reviews of coupled, temperature-dependent contact modelling, identifying the joint treatment of mistuning, friction-parameter uncertainty, and thermal contact-parameter variation as a still-open combination with direct design consequences.**

Keywords—bladed disk mistuning; mode localization; friction damping; under-platform damper; blisk; ring damper; uncertainty quantification; reduced-order modelling

I. INTRODUCTION

The idealized bladed disk analyzed throughout much of the friction-contact modelling literature reviewed in the authors' companion papers is, by construction, cyclically symmetric: every blade is identical, and the assembly's vibratory energy under travelling-wave excitation distributes itself evenly around the disk. No physical bladed disk achieves this exactly. Manufacturing tolerance on aerofoil geometry and material properties, non-uniform wear accumulated in service, and even the small asymmetries introduced by the friction-damping hardware itself all break the assumed cyclic symmetry, a departure the field terms mistuning. The consequence is not merely academic: mistuning can localize vibratory energy onto a small subset of blades, producing peak response amplitudes substantially higher than the tuned-system prediction would suggest, with direct high-cycle-fatigue consequences for exactly the blades unlucky enough to sit at the localization peak.

Friction damping — the shared subject of the authors' companion reviews on contact constitutive modelling, under-platform-damper (UPD) experimental characterization, and thermal-tribological effects — is the primary tool available to a designer for controlling this amplified response, but mistuning and friction do not act independently. A friction interface's effective damping depends on the relative motion it experiences, which mistuning itself alters by redistributing vibratory energy unevenly across the assembly; conversely, friction damping's blade-to-blade energy dissipation can itself suppress or reshape the localization pattern mistuning would otherwise produce. Treating either effect in isolation — mistuning without friction, or friction without mistuning, the implicit assumption in a large fraction of the literature reviewed in the authors' companion papers — risks missing exactly the interaction that determines a real engine's worst-case response.

This review surveys the literature at this specific intersection. Section 2 establishes the fundamentals of bladed-disk mistuning and the reduced-order modelling (ROM) techniques developed to predict its linear consequences efficiently. Section 3 turns to the smaller literature that couples these ROM techniques with the nonlinear friction contact models reviewed in the authors' companion paper, examining what is known about how the two effects interact. Section 4 addresses a case where mistuning and friction interact particularly sharply: integrally bladed disks (blisks), whose monolithic construction eliminates the individually removable blades a conventional UPD requires, forcing designers toward alternative friction-damping concepts. Section 5 surveys the available experimental validation. Section 6 reviews uncertainty-quantification methods for systems where both mistuning and friction-contact parameters are uncertain simultaneously. Section 7 connects this literature explicitly to the authors' companion reviews of coupled, temperature-dependent contact modelling. Section 8 identifies open challenges, and Section 9 concludes.

It is worth noting explicitly where this review sits relative to the two comprehensive external reviews already discussed at length in the authors' companion papers. Krack, Salles and Thouverez's survey of multi-harmonic solution methods for friction-jointed bladed disks [29] and Yuan, Gastaldi, Denimal Goy and Chouvion's comprehensive review of friction-damper modelling, design, and testing [30] each acknowledge mistuning as a factor relevant to friction-damped bladed-disk design, consistent with the mistuning-friction interaction this review documents in depth, but neither treats bladed-disk mistuning theory and reduced-order modelling as a central subject in its own right — that literature is instead the province of Castanier and Pierre's dedicated mistuning review [9], introduced in Section 2.2. The present review's contribution is accordingly the same kind of narrower, more current synthesis the authors' companion papers each provide relative to their own respective landmark reviews: not a replacement for either the mistuning-specific or the friction-specific literature surveys already available, but a focused account of where the two intersect.

II. Fundamentals of Bladed-Disk Mistuning

A. *Sources and Consequences: Mode Localization and Amplitude Amplification*

Ewins' 1969 analysis of detuned bladed-disk vibration [1] is the field's foundational result: even small, random blade-to-blade frequency variations, of the kind manufacturing tolerance alone guarantees, can break the even energy distribution a perfectly cyclic assembly would exhibit under travelling-wave excitation, concentrating vibratory energy instead on a small subset of blades. Figure 1 illustrates this qualitative distinction schematically: a tuned assembly (Fig. 1a) responds with uniform amplitude around the disk, while a mistuned assembly (Fig. 1b) can localize the bulk of that same vibratory energy onto only a few blades, each then experiencing an amplitude substantially above the tuned-system value. Whitehead's widely cited bound [3] quantifies the worst-case severity of this amplification analytically for an idealized model, providing a conservative design estimate long before finite-element-scale mistuned analysis became computationally routine.

Fig. 1. Schematic polar amplitude distribution around a bladed disk: (a) a tuned assembly distributes vibratory response uniformly across all blades; (b) a mistuned assembly can localize the same total vibratory energy onto a small subset of blades, each then experiencing amplitude substantially above the tuned-system value. Distributions are illustrative of the well-documented qualitative phenomenon, not simulation output from a specific bladed-disk model.

This localization phenomenon is not confined to random, unintentional mistuning. Because the same underlying mechanism — breaking cyclic symmetry to redistribute vibratory energy — can in principle be steered deliberately, a parallel design literature treats mistuning as a tool rather than purely a risk, a distinction taken up in Section 2.3.

B. *Reduced-Order Modelling of Mistuned Bladed Disks*

Predicting mistuned response directly from a full finite element model of an entire bladed disk is prohibitively expensive for the repeated, many-configuration analysis mistuning risk assessment requires — the same computational-cost motivation that drove the reduced-order modelling literature reviewed in the authors' companion paper on contact solution strategies. A dedicated family of reduced-order modelling (ROM) techniques has developed specifically for the mistuning problem, summarized in Table 1. Castanier, Ottarsson and Pierre's component-mode-synthesis (CMS) technique [2] remains the most general-purpose approach, assembling a reduced mistuned model from per-sector component modes in a manner directly analogous to the contact-degree-of-freedom reduction reviewed elsewhere in this series. Yang and Griffin's subset-of-nominal-modes (SNM) approach [5] instead represents the mistuned response using only a small subset of the tuned system's own eigenmodes local to the mode family of interest, trading generality for substantial additional speed when only a single mode family's forced response is required. Feiner and Griffin's fundamental mistuning model (FMM) [7], [8] approaches the problem from the opposite direction — not prediction from known blade properties, but identification of the mistuning pattern itself from measured system-level response, a capability of particular value for integrally bladed rotors whose blades cannot be individually removed and bench-tested.

Technique	Reduction basis	Key references	Primary use case
Component mode synthesis (CMS)	Fixed/free-interface component modes per sector, assembled cyclically	Castanier, Ottarsson & Pierre [2]	General-purpose linear mistuned ROM
Subset of nominal system modes (SNM)	A small subset of the full tuned-system's own eigenmodes	Yang & Griffin [5]	Fast forced-response prediction near a mode family
Fundamental mistuning model (FMM)	Isolated mode family + inferred blade-alone frequency perturbations	Feiner & Griffin [7], [8]	Mistuning identification from measured response
Integral / sector mistuning (IMM)	CMS extended to superimpose a sector-level mistuning pattern	Related sector-mistuning extensions of [2]	Industrial FE-scale mistuned rotor models

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TABLE I. PRINCIPAL REDUCED-ORDER MODELLING TECHNIQUES FOR LINEAR MISTUNED BLADED-DISK VIBRATION ANALYSIS.

Castanier and Pierre's 2006 review [9] remains the standard reference synthesizing this ROM literature, alongside the uncertainty-quantification methods Section 6 revisits, and the present review does not attempt to re-derive ground that reference already covers comprehensively. What Castanier and Pierre's review predates, and what forms the specific subject of Section 3, is the subsequent literature that couples these linear mistuning ROMs with the nonlinear friction contact models reviewed in the authors' companion paper — a combination that introduces genuinely new modelling difficulty beyond either effect treated alone.

C. Worst-Case and Intentional Mistuning

Petrov and Ewins directly addressed the design question Whitehead's analytical bound [3] leaves open for a real, finite-element-scale rotor: which specific mistuning pattern, among the enormous combinatorial space of possible blade-to-blade variations, produces the worst-case resonant response for a practical bladed-disk assembly [6]. Their search-based methodology for identifying such worst-case patterns remains a standard reference-point benchmark for subsequent mistuning risk-assessment studies, including several of the uncertainty-quantification methods reviewed in Section 6.

A complementary design philosophy treats mistuning not as a risk to be bounded but as a variable to be exploited. Brewer, Castanier and Pierre examined the free-response consequences of deliberately introduced, harmonically patterned intentional mistuning [4], establishing conditions under which a designed pattern — rather than the random pattern manufacturing tolerance alone produces — can suppress localization or improve aeroelastic stability margins. This intentional-mistuning design philosophy has since been applied specifically to blisks, whose lack of individually replaceable blades removes UPDs as a damping option and correspondingly increases the practical appeal of a vibration-mitigation strategy requiring no separate damping hardware at all, a tension Section 4.3 examines directly.

III. COUPLING MISTUNING WITH NONLINEAR FRICTION CONTACT MODELS

Figure 2 traces the principal milestones bridging the linear mistuning literature of Section 2 and the nonlinear friction-contact literature reviewed in the authors' companion paper, marking where the two threads begin to be treated jointly rather than in isolation. The subsections that follow examine this joint literature in more depth.

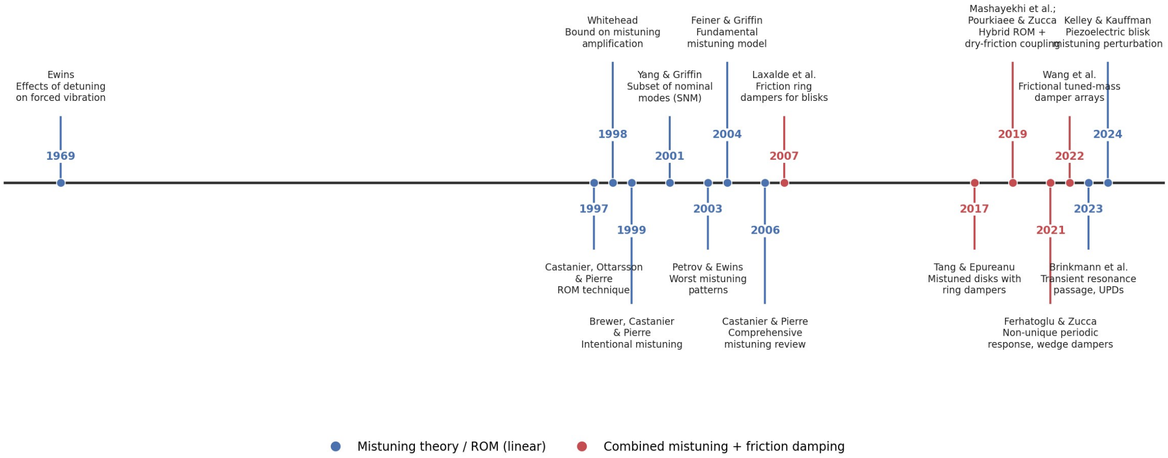

Fig. 2. Timeline of principal developments in linear mistuning theory and reduced-order modelling (blue) alongside developments that couple mistuning explicitly with nonlinear friction damping (red). The combined-treatment literature emerges roughly a decade after the linear ROM techniques of Section 2.2 were established.

A. Reduced-Order Strategies for Combined Mistuning-Friction Systems

Coupling a linear mistuning ROM with a nonlinear friction contact model of the kind reviewed in the authors' companion paper is not a straightforward concatenation of the two: the mistuning ROMs of Table 1 are constructed around the assumption of small, linear perturbations to an otherwise cyclic system, while the MHB–AFT solution methods used for nonlinear friction contact require the full nonlinear force to be evaluated and transformed every iteration, at every blade around the disk. Mashayekhi, Nobari and Zucca addressed this directly with a hybrid reduction scheme retaining enough physical fidelity at each contact interface for the nonlinear force evaluation while still reducing the linear, mistuned portion of the model to a tractable size [13], and Pourkiaee and Zucca developed a comparable reduced-order formulation specifically tailored to shroud-contact mistuned prediction [14]. Zhao, Zhang and Xie combined mistuning and nonlinear friction and contact within a single unified formulation rather than treating the reduction and the nonlinear solve as separate stages [15], and Liu, Liu, Shangguan and Xu applied a three-dimensional microslip contact model within a hybrid frequency-time framework specifically to examine how shroud friction damping modifies the localization pattern mistuning alone would produce [16].

B. How Mistuning and Friction Interact: Key Findings

The consistent qualitative finding across the studies summarized in Table 2 is that friction damping and mistuning-driven localization are neither independent nor simply additive: friction damping measurably narrows, though does not

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eliminate, the amplification band mistuning alone would produce, a relationship sketched schematically in Fig. 4. The underlying mechanism is that friction damping's energy dissipation depends on relative blade-to-blade motion, which mistuning itself modulates by redistributing where in the assembly that relative motion is largest; a friction interface positioned at a blade pair experiencing unusually large mistuning-driven relative motion dissipates proportionally more energy, providing a partial, self-limiting correction to the localization Fig. 1(b) illustrates.

Study (year)	Friction interface modelled	ROM strategy	Key finding
Mashayekhi, Nobari & Zucca [13]	Generic dry-friction contact	Hybrid reduction, mistuned + nonlinear	Full-system nonlinear mistuned analysis at tractable cost
Pourkiaee & Zucca [14]	Shroud friction contact	Reduced-order model, mistuned + shroud	ROM specifically suited to shroud-contact mistuned prediction
Zhao, Zhang & Xie [15]	Nonlinear friction and contact	Mistuned damped-blade formulation	Combined mistuning and nonlinear contact solved jointly
Liu, Liu, Shangguan & Xu [16]	Shroud microslip contact	3D microslip + hybrid frequency-time	Friction damping reduces, but does not eliminate, localization
Tang & Epureanu [12]	Ring damper contact	Mistuned bladed disk + ring damper ROM	Ring-damper damping interacts with mistuning-driven localization
Ferhatoglu & Zucca [18], [19]	Wedge (UPD) contact	Multiple-solution periodic response tracking	Mistuned friction-damped systems admit non-unique steady states

TABLE II. COMPARATIVE SUMMARY OF STUDIES COUPLING BLADED-DISK MISTUNING WITH NONLINEAR FRICTION CONTACT MODELS.

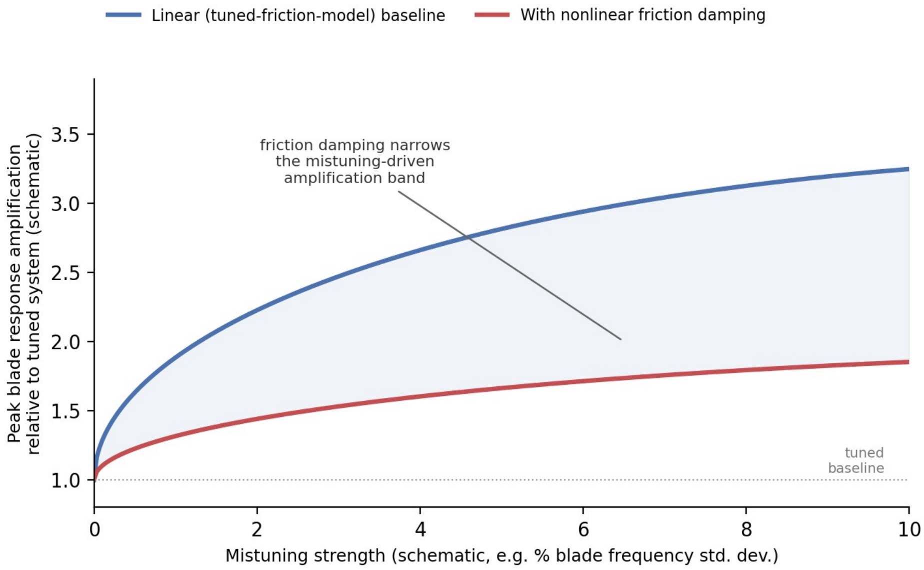

Fig. 4. Schematic relationship between mistuning strength and peak blade response amplification, comparing a linear (friction-damping-absent or tuned-parameter) baseline against a system with nonlinear friction damping active. The narrowing of the amplification band with friction damping present is a qualitative finding consistent across the studies of Table 2; the specific curve shapes shown are illustrative, not digitized data from any single cited study.

Liu, Wang, Li, Xie and Zhang examined this same narrowing effect from a slightly different angle, modelling coupled frictional contact and mistuned variance jointly in a lumped-parameter bladed-disk model specifically to characterize how the two sources of variability shape vibration localization together rather than through friction damping's mitigating effect alone [23], reinforcing that the interaction sketched in Fig. 4 is a genuinely two-way coupling rather than friction simply acting as a one-directional correction applied on top of an otherwise unchanged mistuning pattern.

Tang and Epureanu examined this interaction specifically for ring dampers — the friction-damping concept most relevant to blisks, discussed further in Section 4.2 — finding that ring-damper damping and mistuning-driven localization interact in a manner sensitive to where, circumferentially, the ring damper's own effective stiffness and damping happen to align with the mistuning pattern's localization peak [12], underscoring that the narrowing effect illustrated schematically in Fig. 4 is a general qualitative tendency rather than a guaranteed, geometry-independent outcome.

C. Non-Uniqueness and Response Bounds Under Combined Uncertainty

A further complication specific to the combined mistuning-friction problem, absent from either linear mistuning analysis or single-blade nonlinear friction-contact analysis treated separately, is that the coupled nonlinear system can admit multiple distinct steady-state periodic solutions at a single excitation frequency. Ferhatoglu and Zucca demonstrated this non-uniqueness explicitly for wedge-damper-equipped bladed disks [18], and developed systematic methods for computing the resulting response bounds — the envelope of possible steady-state amplitudes across all admissible solutions, rather than a single deterministic curve — that a mistuned, friction-damped bladed disk can exhibit [19]. This non-uniqueness result reinforces a point already raised in the authors' companion review of contact constitutive modelling regarding path-dependent, history-sensitive contact behaviour: a mistuned, friction-damped bladed disk's response is not fully determined by its excitation frequency and amplitude alone, and a forced-response prediction that reports only a single curve may understate the range of physically admissible outcomes. Butlin, Spelman, Ghaderi, Midgley and Umehara approached a closely related question from a design-oriented angle, developing methods to predict response bounds directly under uncertain friction coupling rather than tracing every individual admissible solution branch [17], a framing that connects naturally to the broader uncertainty-quantification literature reviewed in Section 6.

D. Computational Cost of Combined Analysis

Yuan, Scarpa, Allegri, Titurus, Patsias and Rajasekaran's comprehensive review of computational techniques for mistuning analysis [31] is, alongside Castanier and Pierre's review [9] already discussed in Section 2.2, a standard reference for the linear mistuning literature — and, notably, explicitly excludes nonlinearity from its scope, stating the nonlinear problem as beyond the review's treatment. This is not an oversight but an accurate reflection of where the computational difficulty compounds: a stochastic mistuning study requires many independent realizations (Monte Carlo sampling being the most direct, if costly, approach) to characterize the resulting response distribution, and coupling each realization with a nonlinear friction-contact solve of the kind reviewed in the authors' companion paper multiplies, rather than adds to, the already substantial cost of either treated alone. Capiez-Lernout and Soize [33] quantified this multiplicative structure directly for a geometrically nonlinear, mistuned blisk, noting explicitly that the product of the number of detuned configurations and the number of Monte Carlo realizations required for statistical convergence renders direct nonlinear solution intractable at the scale industrial risk assessment would require, motivating a probabilistic surrogate-modelling approach in its place — the same computational-cost motivation, applied to the mistuning axis specifically, that motivates the surrogate and reduced-order approaches reviewed throughout the authors' companion papers for the friction-contact axis alone.

Tang, Baek and Epureanu [32] addressed this compounding cost directly for blisks specifically, developing reduced-order models spanning both small and large mistuning regimes in combination with friction dampers, structured so that the same reduction basis remains valid whether the mistuning perturbation is small (permitting linear perturbation methods) or large (requiring the fuller nonlinear treatment), avoiding the need to rebuild the reduced-order model from scratch as mistuning severity varies across a design or uncertainty study. This adaptive-basis strategy is, in the authors' view, a template worth extending to the thermally-varying contact parameters reviewed in the authors' companion paper: just as mistuning severity need not be fixed in advance for a well-constructed ROM to remain valid, neither should the operating temperature at which a coupled contact model's parameters are evaluated.

IV. Friction Damping Strategies for Blisks

A. The Blisk Constraint: No Removable Blades

Integrally bladed disks (blisks) — machined or additively manufactured as a single monolithic component rather than assembled from individually attached blades — offer substantial weight and part-count advantages over conventional bladed-disk construction, and have accordingly become standard in modern compressor and, increasingly, turbine stages. Figure 3 illustrates the structural consequence this construction choice has for friction damping specifically: a conventional bladed disk (Fig. 3a) has individually removable blades, between whose platforms a UPD of the kind reviewed extensively in the authors' companion paper can be installed as a discrete, replaceable component. A blisk (Fig. 3b) has no such interface — the blade root, platform, and disk are a single continuous piece of material, eliminating the removable-blade UPD installation path entirely.

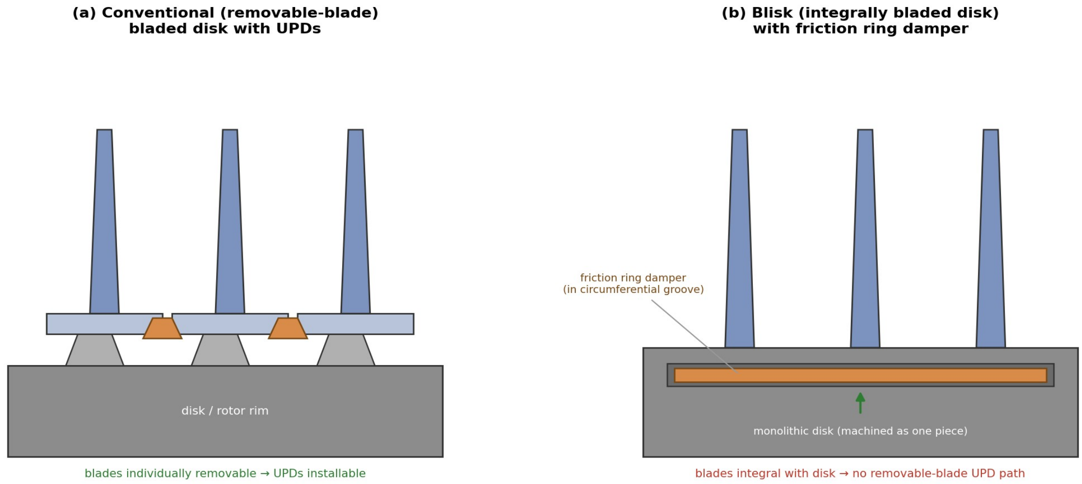

Fig. 3. Structural comparison between (a) a conventional bladed disk with individually removable blades, permitting UPD installation between adjacent platforms, and (b) a blisk (integrally bladed disk), whose monolithic construction eliminates the removable-blade interface a conventional UPD requires, motivating alternative friction-damping concepts such as the friction ring damper shown installed in a circumferential groove beneath the blisk rim.

This constraint is precisely why the mistuning-friction literature reviewed in Section 3 matters disproportionately for blisks specifically: because blisks cannot fall back on the well-characterized UPD damping strategy reviewed in the authors' companion paper, they are simultaneously more exposed to mistuning-driven localization risk (since blisk manufacturing processes, while producing very low mistuning on average, still leave the small residual variation Section 2.1 shows is sufficient to localize response) and more dependent on the alternative friction-damping concepts surveyed in Section 4.2, whose interaction with that mistuning is correspondingly more consequential to characterize correctly.

It is also worth noting that the manufacturing processes increasingly used to produce blisks — five-axis machining from solid billet, and, for an expanding range of applications, additive manufacturing — do not merely constrain which friction-damping concepts remain available; they also shape the residual mistuning distribution Section 2.1 shows is sufficient to localize response in the first place. Additively manufactured blisks in particular can exhibit systematic, rather than purely random, geometric variation tied to build orientation and thermal history during manufacture, a source of mistuning structurally different from the independent, blade-to-blade random variation the classical mistuning literature of Section 2 was originally developed to characterize. Whether the reduced-order and uncertainty-quantification methods reviewed in Sections 2 and 6 — built around an implicit assumption of independent or weakly correlated blade-to-blade variation — remain adequate for the more structured, build-orientation-correlated mistuning an additively manufactured blisk may exhibit is not, to the authors' knowledge, directly addressed in the literature reviewed here.

B. *Friction Ring Dampers and Tuned-Mass Arrays*

The friction ring damper, sketched in Fig. 3(b), is the most established alternative: a ring, held in a circumferential groove beneath the blisk rim by centrifugal load, dissipates energy through relative sliding against the groove wall as the disk vibrates. Laxalde, Thouverez, Sinou and Lombard developed the foundational lumped-parameter model and MHB–AFT solution approach for this configuration [10], reporting that damping effectiveness depends strongly on the level of dynamic coupling between blade and disk motion — a design-sensitive dependency with no direct analogue in the UPD literature, where the damper couples two adjacent blades directly rather than each blade to a shared ring. Laxalde, Thouverez and Lombard extended this analysis to full integrally bladed disk finite element models using a dynamic Lagrangian frequency-time method adapted to cyclic structures [11], confirming the coupling-level sensitivity at industrial scale and providing design guidance for ring geometry and preload.

More recently, Wang, Yang, Ji, Bi and Zheng proposed a frictional tuned-mass-damper array — multiple discrete frictional oscillators, each tuned to a specific mode family of interest — as a multi-mode alternative to a single ring damper's inherently narrower effective bandwidth [20], reporting that the array approach remains robust against small deviations in individual oscillator tuning, a practically important property given the manufacturing tolerance considerations already central to the mistuning discussion of Section 2. Table 3 summarizes these and the remaining strategies discussed in this section against a common set of criteria.

Strategy	Mechanism	Key references	Design trade-off
Friction ring damper	Centrifugally loaded ring in a circumferential groove beneath the blisk rim	Laxalde et al. [10], [11]	Effective but couples strongly to disk-blade dynamic coupling level
Frictional tuned-mass damper array	Multiple frictional oscillators tuned to specific mode families	Wang, Yang, Ji, Bi & Zheng [20]	Multi-mode coverage at the cost of added hardware complexity
Intentional mistuning (mass/stiffness pattern)	Deliberately patterned blade-to-blade variation, no separate damping hardware	Brewer, Castanier & Pierre [4]	No added parts, but design-specific and less robust to off-design conditions
Piezoelectric shunt / active perturbation	Electromechanically coupled stiffness perturbation, testable in software	Kelley & Kauffman [22]	Rapid reconfigurable testing; coupling must exceed inherent mistuning to matter

TABLE III. COMPARATIVE SUMMARY OF FRICTION-DAMPING AND VIBRATION-MITIGATION STRATEGIES APPLICABLE TO BLISKS, WHICH CANNOT ACCOMMODATE A CONVENTIONAL REMOVABLE-BLADE UNDER-PLATFORM DAMPER.

C. *Intentional Mistuning as a Complementary Strategy*

Because a blisk's monolithic construction rules out the removable-blade UPD path but not deliberate variation of blade geometry, mass, or local stiffness during design and manufacture, intentional mistuning — introduced in Section 2.3 as a design tool rather than purely a risk — holds particular appeal for blisks specifically, since it requires no additional damping hardware at all. Kelley and Kauffman's piezoelectric-induced stiffness-perturbation methodology [22] addresses a practical obstacle to developing and validating such intentionally mistuned blisk designs: because a single physical blisk specimen can only ever represent one mistuned configuration, and because advanced manufacturing processes now routinely produce blisks with very low inherent mistuning, testing a range of candidate intentional-mistuning patterns experimentally would otherwise require an impractical number of physical specimens. By using electromechanically coupled piezoelectric elements to impose a software-configurable stiffness perturbation on a single specimen, their approach enables rapid experimental screening of candidate intentional-mistuning patterns, provided the imposed electromechanical coupling is large enough to dominate the specimen's own small inherent mistuning — a calibration requirement the authors report is non-trivial to satisfy for the very-low-mistuning blisks representative of current manufacturing practice.

It is worth noting explicitly that intentional mistuning and friction damping are not mutually exclusive strategies even for a blisk that could, in principle, accommodate a ring damper: nothing in the reviewed literature precludes combining a deliberately patterned intentional-mistuning design with a friction ring damper simultaneously, and Table 3's four strategies should accordingly be read as a design-space menu rather than a set of mutually exclusive alternatives. Whether such a combined strategy outperforms either approach alone — and specifically whether the localization-narrowing interaction Fig. 4 illustrates for random mistuning also holds for a deliberately patterned intentional-mistuning design — is not, to the authors' knowledge, addressed directly in the literature reviewed here, and is identified as an open question in Section 8.

V. Experimental Validation of Combined Mistuning-Friction Behaviour

Experimental validation specifically of the combined mistuning-friction interaction discussed in Section 3 is considerably sparser than the room-temperature UPD rig literature reviewed in the authors' companion paper, in large part because isolating mistuning's contribution from friction's contribution requires either a specimen with precisely characterized, repeatable blade-to-blade variation or a rig capable of testing both a tuned and a matched mistuned configuration under otherwise identical conditions. Usmanov, Battiato, Firrone, Conte, Rosso and D'Ettole addressed this directly with an academic demonstrator specifically designed to separate intrinsic mistuning (inherent to the manufactured specimen) from contact mistuning (introduced by the UPDs themselves, whose individual contact conditions are never perfectly identical from damper to damper), tested on a 48-blade disk in a vacuum spinning rig using blade tip-timing measurement [24]. Their results confirm, at rotating, multi-blade experimental scale, the qualitative finding already established computationally in Section 3.2: friction damping measurably reduces, without eliminating, the vibration localization mistuning alone produces, and—notably—the UPDs' own contact-to-contact variability is itself a non-negligible source of effective mistuning distinct from the blades' intrinsic geometric and material variation.

Brinkmann, Hoffmann, Panning-von Scheidt and Stüer [21] extended experimental validation beyond steady-state forced response to the transient resonance-passage behaviour more representative of actual engine start-up and shutdown, measuring a free-standing turbine blisk's response with and without UPDs installed and confirming that friction damping is able to suppress transient ring-down amplification just as it suppresses steady-state localization amplification, with geometric mistuning again shown to produce different transient responses across otherwise nominally identical blades. Taken together, these two studies represent, at the time of writing, the most direct available experimental evidence that the mistuning-friction interaction established computationally in Section 3 persists under rotating, multi-blade, and transient conditions — though both studies stop short of the elevated-temperature conditions the authors' companion review of thermal-tribological effects shows also matter, a gap Section 7 returns to.

VI. Uncertainty Quantification for Coupled Mistuning-Friction Systems

Sections 2–5 establish that mistuning, friction, and their interaction are each subject to substantial parametric uncertainty individually — mistuning through manufacturing tolerance and wear, friction-contact parameters through the identification variability the authors' companion review of UPD experimental characterization documents extensively. Treating a mistuned, friction-damped bladed disk deterministically, using a single nominal value for every mistuning and contact parameter, therefore risks understating the true range of response the assembly can exhibit. Figure 5 sketches the resulting distinction schematically: a deterministic, nominal-parameter prediction (the blue curve) sits inside, rather than at the edge of, the envelope a full parametric uncertainty treatment reveals, with the worst-case envelope bound (the dashed line) tracing the upper edge of response amplitudes achievable across the admissible parameter range.

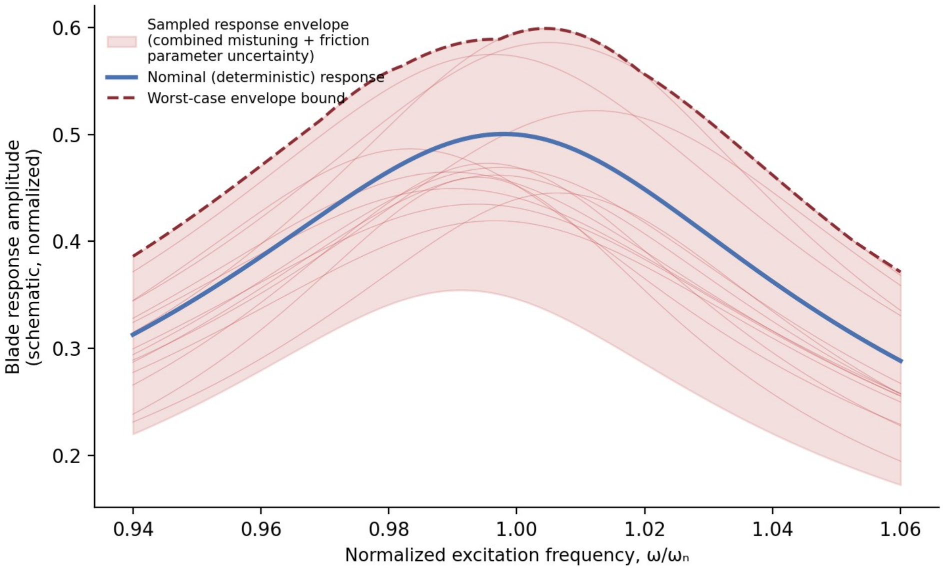

Fig. 5. Schematic frequency-response envelope illustrating the distinction between a deterministic, nominal-parameter prediction and the response band revealed by sampling across combined mistuning and friction-contact parameter uncertainty. Individual thin lines represent illustrative sampled realizations; the dashed envelope traces the resulting worst-case bound. Curves are schematic, not simulation output from a specific bladed-disk model.

Munachi, Fu, Zhao and colleagues' recent critical review [25] surveys the resulting uncertainty-quantification methodology comprehensively for mistuned bladed disks specifically — spanning Monte Carlo simulation, polynomial chaos expansion, Gaussian-process and neural-network surrogates, and random-field representations of spatially correlated blade-to-blade variation — and the present review does not attempt to duplicate that survey's breadth. What is

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worth highlighting specifically, because it connects directly to the friction-parameter uncertainty already discussed at length in the authors' companion review of UPD experimental characterization, is that the same polynomial-chaos and Sobol-sensitivity methodology reviewed there for friction-contact-parameter uncertainty alone — in the work of Yuan, Fantetti, Denimal, Bhatnagar, Pesaresi, Schwingshackl and Salles [27] and of Yuan, Jones, Setchfield and Schwingshackl [28] — is directly extensible, at least in principle, to a combined mistuning-and-friction uncertainty treatment of the kind Fig. 5 illustrates, though neither of those two studies incorporates mistuning explicitly. Yang, She, Wu, Mi, Zang and Li's multi-source uncertainty framework for UPD-equipped blades [26] comes closest among the UPD-specific literature reviewed elsewhere in this series to the combined treatment this section calls for, propagating manufacturing, excitation, and wear-state uncertainty jointly, though again without an explicit mistuning axis of variation distinct from the UPD contact-parameter variability itself.

The practical conclusion this section draws is that the individual methodological pieces required for a genuinely joint mistuning-friction-thermal uncertainty quantification already exist, distributed across the mistuning-specific literature synthesized by Munachi et al. [25] and the friction-parameter-specific literature reviewed in the authors' companion paper, but have not yet been assembled into a single combined treatment — a gap structurally analogous to the constitutive-modelling gap identified in the authors' companion review of thermal-tribological effects, where the necessary tribological and structural-dynamics ingredients likewise exist independently but have not been combined.

It is worth being specific about what assembling these pieces would practically require, since the answer is not simply running three existing uncertainty-quantification pipelines in sequence. A mistuning realization is typically drawn from a random field over blade-to-blade properties; a friction-parameter realization is typically drawn from an identification-uncertainty distribution of the kind the authors' companion review of UPD experimental characterization documents; and a thermal-parameter realization would, following the calibration strategy the authors' companion review of thermal-tribological effects proposes, be drawn from a distribution over the fitted glaze-layer transition temperature and related coefficients. A joint uncertainty propagation would need to sample all three simultaneously, propagate each combination through a full nonlinear, mistuned, temperature-indexed MHB–AFT solve, and do so enough times for the resulting response statistics to converge — precisely the multiplicative cost structure Section 3.4 already shows renders the two-way mistuning-friction problem expensive even before a thermal axis is added. Surrogate-modelling strategies of the kind reviewed in Section 3.4 and in the authors' companion papers are accordingly not an optional convenience for this three-way problem but very plausibly a practical necessity.

VII. Implications for Coupled, Temperature-Dependent Contact Models

The authors' companion reviews establish two further points this review's findings connect to directly. First, the review of dry-friction contact constitutive modelling shows that normal–tangential coupled contact models materially affect resonance-amplitude prediction relative to uncoupled models, at least at room temperature; whether this coupling effect itself interacts with mistuning-driven localization in the manner Section 3.2 shows friction damping generally does is, to the authors' knowledge, unexamined in the published literature — every mistuning-friction study summarized in Table 2 uses an uncoupled or normal-load-variation contact law rather than the coupled formulations reviewed in the authors' companion paper. Second, the review of thermal-tribological effects shows that friction and contact-stiffness parameters are themselves temperature-dependent; because Table 3 shows blisk friction-damping strategies specifically (ring dampers, tuned-mass arrays) rely on the same class of dry-friction contact whose temperature dependence that review documents, the thermally-blind treatment already flagged as a limitation for UPDs applies with at least equal force to the blisk damping strategies reviewed in Section 4.2.

Combining all three threads — mistuning, coupled normal–tangential contact behaviour, and temperature dependence — into a single predictive framework is, on the evidence reviewed across this series, not yet attempted anywhere in the published literature. This is not a criticism of any individual study; each of the three threads is independently well-developed and individually validated, as the authors' companion reviews and the present review each document in turn. It is, however, precisely the combination a real engine's operating envelope requires: a rotating, mistuned, friction-damped bladed disk operating across a substantial temperature range, for which no published contact model, reduced-order formulation, or uncertainty-quantification framework yet accounts for all three factors simultaneously.

VIII. Open Challenges and Future Directions

A. Mistuning-Coupling Interaction

Section 7 already identified the most direct open question this review raises: whether the room-temperature significance of normal–tangential contact coupling, established for a tuned two-blade UPD rig in the authors' companion review, persists, strengthens, or diminishes once mistuning is introduced. Because coupling and mistuning each independently redistribute contact force and relative displacement around the disk — coupling through the lift-up mechanism at each individual contact, mistuning through uneven blade-to-blade energy distribution — there is no a priori reason to expect the two effects to be simply additive, and Section 3.3's finding of non-unique steady-state solutions under mistuning-friction coupling alone suggests the combined coupling-mistuning problem could admit comparably rich, currently uncharacterized behaviour. A first, tractable step toward this question would not require a full mistuned-disk study: repeating the two-blade, room-temperature validation already available for the coupled contact model, but with the two blades deliberately detuned by a small, controlled amount, would establish whether coupling significance is sensitive to even the simplest possible mistuning pattern before a full stochastic mistuning study is attempted.

B. Thermally-Aware Blisk Damping Strategies

None of the blisk friction-damping strategies summarized in Table 3 has been evaluated, in the literature reviewed here, against the temperature-dependent contact parameters the authors' companion review of thermal-tribological effects documents. Because ring dampers and tuned-mass-damper arrays rely on the same glaze-layer-mediated friction and stiffness mechanisms reviewed there, and because blisks' typical application in the hotter, more highly loaded stages of modern engines makes the thermal envelope these components experience at least as demanding as that of UPD-equipped stages, this represents a direct, currently unaddressed extension of both this review's and the authors' thermal-tribological review's findings.

C. Joint Uncertainty Quantification Across All Three Axes

Section 6 identified the methodological pieces needed for a joint mistuning-friction-thermal uncertainty treatment as individually available but not yet combined. Assembling them is not merely a matter of running existing methods together: mistuning uncertainty, friction-parameter uncertainty, and thermal-parameter uncertainty are not necessarily independent of one another — wear-induced mistuning, for instance, plausibly correlates with the same accumulated sliding history the authors' companion thermal review shows also drives glaze-layer state — and a naive joint propagation that treats all three as statistically independent could over- or under-state the resulting response envelope depending on the true correlation structure, which has not, to the authors' knowledge, been characterized for any bladed-disk system.

D. Validated Combined Design Strategies for Blisks

Section 4.3 noted that intentional mistuning and friction damping are not mutually exclusive strategies for a blisk capable of accommodating both, yet no study reviewed here evaluates a combined intentional-mistuning-plus-ring-damper design against either strategy alone. Given that Kelley and Kauffman's piezoelectric perturbation methodology [22] now makes rapid experimental screening of candidate intentional-mistuning patterns practical, and that the ring-damper and tuned-mass-damper-array literature of Section 4.2 provides mature friction-damping design tools, the experimental and computational infrastructure needed to evaluate such combined strategies appears to already exist; what is missing is a study that brings the two together.

IX. CONCLUSION

Bladed-disk mistuning and friction damping have each been studied extensively, and largely separately, over more than five decades: mistuning's mode-localization and amplification consequences have been characterized analytically, computationally, and experimentally since Ewins' foundational 1969 analysis, and dry-friction contact damping has been the subject of the extensive constitutive-modelling and experimental-characterization literature reviewed in the authors' companion papers. The smaller literature reviewed here, coupling the two explicitly, establishes a clear and consistent qualitative finding: friction damping narrows, without eliminating, the response amplification mistuning alone produces, through a mechanism — friction dissipation scaling with the same relative motion mistuning redistributes — that makes the two effects genuinely interactive rather than independently additive. This interaction matters disproportionately for blisks specifically, whose monolithic construction eliminates the conventional under-platform-damper path and correspondingly increases reliance on the alternative friction-ring-damper, tuned-mass-array, and intentional-mistuning strategies reviewed in Section 4.

What remains open, on the evidence reviewed across this series of papers, is the joint treatment of mistuning alongside the coupled, temperature-dependent contact behaviour the authors' companion reviews establish as independently significant. No published study combines mistuning, normal–tangential contact coupling, and thermal contact-parameter dependence within a single predictive or uncertainty-quantification framework, despite each pairwise combination — mistuning with friction, friction with temperature, and, implicitly, mistuning with the manufacturing and wear processes that also drive thermal contact variability — being independently documented as consequential. Closing this three-way gap, building outward from the coupled, temperature-dependent contact model the authors' companion reviews propose, is identified here as a natural and, in the authors' view, necessary extension of the present research programme.

A final, practical note follows directly from Section 3.4's discussion of computational cost. Any research programme that attempts the three-way combination this review calls for will inherit the multiplicative cost structure already documented for mistuning and friction treated pairwise: a thermally indexed sweep, layered on top of a stochastic mistuning study and a nonlinear friction solve, multiplies rather than adds to an already substantial computational burden. The reduced-order and surrogate-modelling strategies reviewed throughout this series — adaptive-basis reduction spanning small and large mistuning [32], probabilistic-learning surrogates for detuning optimization [33], and the physics-informed and multi-coupled neural-network surrogates reviewed in the authors' companion papers — are not incidental conveniences but a practical precondition for the combined analysis this review identifies as the field's next necessary step; without them, the three-way problem this review poses would likely remain computationally intractable even where it is conceptually well posed.

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