

Distributional FKF–FKL Spectral Framework for Resilient Smart Logistics with Digital Twins, Sustainable Mobility, and Quantum-Constrained Dispatch

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Abstract— This reconstructed study develops an integrated mathematical and operational architecture in which distributional integral transforms provide the analytical layer for resilient smart logistics. The framework retains the original testing-function-space construction, its seminorm topology, Fréchet-space completeness, dual-space characterization, fractional reaction–diffusion formulation, Mittag-Leffler representation, and Talbot inversion. Around this mathematical core, the study adds a genuine logistics interpretation through two-sided FKF/FKL spectral analysis, sequential convergence, time-shift and exponential-modulation behavior, residue-based resonance extraction, and shift-invariant analysis of multi-echelon lead times. The resulting spectral state is connected to digital-twin telemetry, Internet of Things sensing, green logistics indicators, blockchain traceability with hash-chain provenance, Industry 5.0 human-centric automation, swipe authorization, anti-theft signaling, hybrid-electric mobility, and quantum-annealing QUBO models for constrained Unit Load Device dispatch. The objective is not to replace the underlying equations but to show how the same distributional machinery can support heterogeneous physical and cyber-physical supply-chain signals while preserving mathematical consistency.

Keywords: *distributional FKF transform; FKL transform; testing-function space; sequential convergence; time-shift property; exponential modulation; residue-based resonance; shift-invariant spectral analysis; multi-echelon lead time; supply chain resilience; digital twin; green logistics; blockchain traceability; hash-chain provenance; quantum logistics; quantum annealing; QUBO; Unit Load Device; smart logistics; Internet of Things; intelligent transportation systems; Industry 5.0; hybrid-electric vehicle; battery sizing; solar-assisted electric mobility.*

I. INTRODUCTION

Modern supply chains behave as coupled cyber-physical systems rather than isolated transportation or inventory processes. Their state is distributed across sensors, vehicles, warehouses, energy stores, authorization devices, digital twins, and optimization engines [33]–[36]. The present reconstruction therefore treats mathematical transformation, operational sensing, and constrained decision-making as successive layers of one architecture.

The original manuscript establishes a functional-analytic basis for generalized distributional transforms and connects it with quantum algorithms, electrochemical dynamics, smart mobility, and supply-chain resilience [7]–[12], [31]–[36]. The expanded architecture interprets the FKF/FKL family as a shift-aware spectral interface. Fourier structure represents temporal oscillation and recurring lead-time behavior, while the Kontorovich–Lebedev component supplies a scale-sensitive representation for positive operational variables [11], [17].

A resilient supply chain must preserve feasible operation when lead times, demand, energy availability, vehicle state, authorization status, or network connectivity deviate from nominal values [10], [23], [24]. The framework therefore separates sensed state, transformed state, constraint state, and decision state. Green logistics introduces energy and emissions constraints, while blockchain traceability adds tamper-evident provenance [27], [28]. Industry 5.0 human-centric automation retains authorization and intervention points [21], [33].

II. WEIGHTED TESTING SPACES FOR DISTRIBUTIONAL LOGISTICS SIGNALS

The testing-function space is the mathematical entry point for signals whose derivatives, growth rates, and boundary behavior must be controlled simultaneously. In logistics, such signals

may represent demand pulses, lead-time trajectories, battery-current profiles, telemetry streams, or event-driven authorization records [12].

A. Weighted Regularity and Admissible Growth

Let the open operational domain be a subset of real n -space. The testing function space consists of infinitely differentiable complex-valued functions subject to weighted growth conditions.

Definition 2.1 (The space Φ). Let α and β be fixed structural parameters. For every pair of multi-indices k, m we define

$$\gamma_{k,m}(\varphi) = \sup_{x \in \Omega} \left| x^k \partial^m (w_{\alpha,\beta}(x) \varphi(x)) \right| \quad (1)$$

where the multi-index operator has order $|m|$. The space consists of all smooth test functions for which every seminorm remains finite.

$$\Phi = \left\{ \varphi \in C^{\infty(\Omega)} : \gamma_{\{k,m\}}(\varphi) < \infty \text{ for all multi-indices } k, m \right\} \quad (2)$$

An admissible weight may be taken in product form

$$w_{\alpha,\beta}(x) = (1 + |x|)^\alpha \exp(\beta|x|) \quad (3)$$

Large weights penalize states that grow too rapidly, while derivative orders capture detailed fluctuations. This is useful for multi-echelon lead times because small oscillations can be separated from large structural deviations [11], [14].

B. Seminorm Control and Operational Meaning

Proposition 2.2. For each fixed pair (k, m) , the functional is a seminorm on Φ . Absolute homogeneity reads

$$\gamma_{k,m}(\lambda\varphi) = |\lambda| \gamma_{k,m}(\varphi) \quad (4)$$

and the triangle inequality is

$$\gamma_{k,m}(\varphi + \psi) \leq \gamma_{k,m}(\varphi) + \gamma_{k,m}(\psi) \quad (5)$$

Non-negativity corresponds to measurable deviation magnitude, absolute homogeneity supports rescaling across sensors, and the triangle inequality allows independent perturbations to be aggregated in digital-twin telemetry [11], [26].

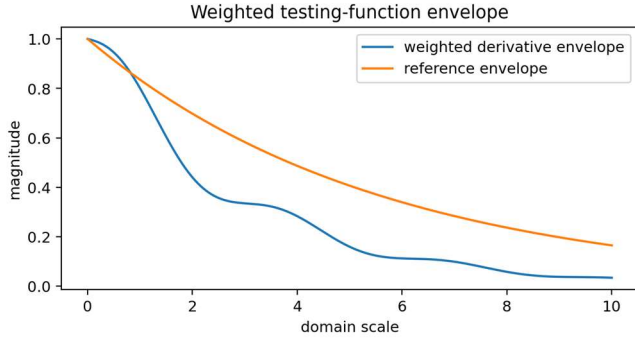


Fig. 1. Weighted testing-function envelope illustrating controlled growth and derivative-sensitive admissibility.

III. TOPOLOGICAL COMPLETENESS, DUALITY AND SEQUENTIAL CONVERGENCE

A transform framework is useful only if convergence is meaningful. The Frechet structure supplies a complete metric topology generated by countably many seminorms, while the dual space supplies continuous linear functionals capable of representing generalized signals [12].

A. Frechet Structure and Convergence

Theorem 3.1. Endowed with the topology generated by the countable collection of seminorms, the testing function space is a Frechet space. Enumerating the seminorms as p_n , define

$$d(\varphi, \psi) = \sum_{n=1}^{\infty} 2^{-n} \frac{p_n(\varphi - \psi)}{1 + p_n(\varphi - \psi)} \quad (6)$$

If a sequence is Cauchy with respect to d , then every seminorm is Cauchy. Completeness of smooth functions under uniform convergence of derivatives on compact subsets yields a limit in the testing space. Sequential convergence has a direct digital-twin interpretation: a stream of telemetry snapshots is treated as a sequence in the testing space, provided every required seminorm stabilizes [12], [20].

B. Dual-Space Representation of Singular Events

Theorem 3.2. A linear functional T belongs to the dual if and only if there exist $C > 0$ and N such that

$$|\langle T, \varphi \rangle| \leq C \max_{\{|k|+|m| \leq N\}} \gamma_{k,m}(\varphi) \quad (7)$$

A sudden shipment interruption, sensor spike, battery-current impulse, or authorization transition may not behave like an ordinary smooth function. A continuous functional on the testing space represents such events while remaining compatible with transform operations [12], [31], [33].

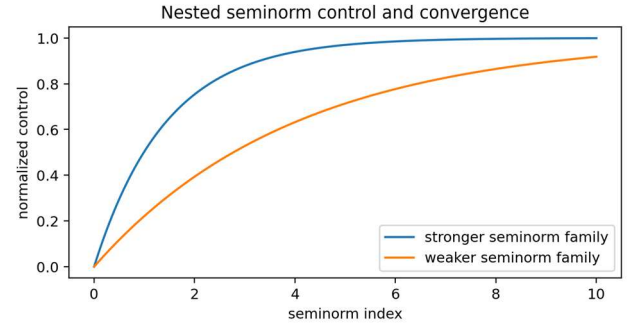


Fig. 2. Nested seminorm control: stronger and weaker seminorm families illustrate how convergence is monitored at multiple scales.

IV. DISTRIBUTIONAL FKF/FKL SPECTRAL ANALYSIS FOR MULTI-ECHELON DYNAMICS

The Fourier factor resolves temporal oscillations, periodic demand, and recurring lead-time cycles; the Kontorovich–Lebedev factor, through Macdonald-function behavior, provides a positive-scale representation suited to intensity, distance, and other non-negative operational variables [17].

A. Two-Sided Transform Architecture

The distributional two-sided FKF action on an admissible generalized signal is

$$\mathcal{F}(f)(\tau, \nu) = \langle f, K_{\tau, \nu} \rangle \quad (8)$$

with kernel structure involving the Macdonald function of imaginary order

$$K_{\tau, \nu}(t, r) = \frac{e^{\{-i\tau t\}}}{\sqrt{(2\pi)}} \sqrt{r} K_{i\nu}(r) \quad (9)$$

The time-shift property compares lead-time profiles that begin at different operational origins [18]:

$$\mathcal{F}\{f(\cdot - t_0)\}(\tau, \nu) = e^{\{-i t_0 \tau\}} \mathcal{F}\{f\}(\tau, \nu) \quad (10)$$

Exponential modulation provides controlled growth or attenuation [19]:

$$\mathcal{F}\{e^{\{at\}}f\}(\tau, \nu) = \mathcal{F}\{f\}(\tau + ia, \nu) \quad (11)$$

Differentiation in the original variable corresponds to polynomial multipliers in the spectral variable [16]:

$$\mathcal{F}\{\partial_t f\}(\tau, \nu) = i\tau \mathcal{F}\{f\}(\tau, \nu) \quad (12)$$

B. Resonance and Multi-Echelon Lead-Time Interpretation

A compact operational spectral model is a superposition of shifted modes with echelon-dependent amplitudes and phases:

$$L(t) = \sum_{\ell=1}^M A_{\ell} \cos(\omega_{\ell} t + \theta_{\ell}) \quad (13)$$

Residue-based resonance extraction isolates dominant poles of the transformed representation [13]:

$$\text{Res}_{\{\zeta=\zeta_{\ell}\}(\mathcal{F}\{L\}(\zeta))} = \frac{1}{2\pi} \int_{\Gamma_{\ell}} \mathcal{F}\{L\}(\zeta) d\zeta \quad (14)$$

In a multi-echelon network, resonance can indicate persistent replenishment cycles, synchronization between facilities, or recurrent bottlenecks [14], [15]. Battery-management constraints define admissible physical states around these modes [7], [32].

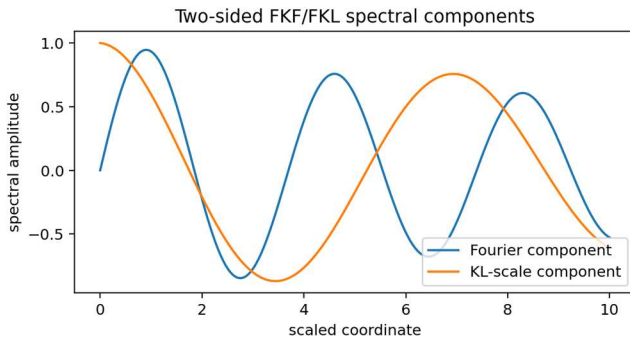


Fig. 3. Conceptual decomposition of a logistics signal into temporal Fourier structure and positive-scale KL structure.

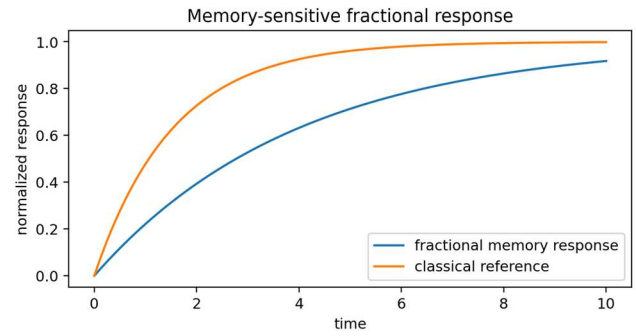


Fig. 4. Illustrative memory-sensitive response: fractional behavior evolves differently from a classical first-order reference.

V. FRACTIONAL MEMORY, NON-LOCAL TRANSPORT AND ELECTROCHEMICAL DYNAMICS

Fractional models are useful when the current state depends on accumulated history rather than only an instantaneous derivative. This is relevant to diffusion, battery degradation, delayed transport, and resilience recovery [7], [11].

A. Distributional Reaction–Diffusion Formulation

Consider the non-local time-fractional reaction-diffusion equation

$$\partial_t^\alpha u(x, t) = \mathcal{L}_x u(x, t) + R(u) + f(x, t) \quad (15)$$

subject to admissible boundary conditions and $u(x, 0) = u_0(x)$. The Caputo derivative of order α in $(0, 1]$ is

$$\partial_t^\alpha u = \frac{\int_0^t (t-s)^{-(\alpha)} \partial_s u(x, s) ds}{\Gamma(1-\alpha)} \quad (16)$$

Solar contribution may enter as an additional energy inflow in the source term [9], [22].

B. Mittag-Leffler Response and Numerical Inversion

The Laplace transform in time converts (15) into an algebraic modal problem. For the Caputo derivative,

$$\mathcal{L}\{\partial_t^\alpha u\}(s) = s^\alpha \tilde{u}(s) - s_0^{\{\alpha-1\}} u \quad (17)$$

Expanding over spatial eigenfunctions and inverting with the two-parameter Mittag-Leffler kernel gives

$$\mathcal{L}^{\{-1\}} \left\{ \frac{s^{\{\alpha-1\}}}{s^\alpha + \lambda} \right\} = E_\alpha(-\lambda t^\alpha) \quad (18)$$

The closed-form modal solution is

$$u(x, t) = \sum_{n=1}^{\infty} c_n E_\alpha(-\lambda_n t^\alpha) \varphi_n(x) + u_{part} \quad (19)$$

When a closed form is unavailable, numerical inversion along a modified Talbot contour is used:

$$u(t) \approx \frac{1}{2\pi} \int_{\Gamma_T} e^{st} \tilde{u}(s) ds \quad (20)$$

Sequential convergence supplies a criterion for stable reconstruction from observation sequences [12], and residue extraction isolates repeated spectral modes [13].

VI. DIGITAL-TWIN, GREEN LOGISTICS AND BLOCKCHAIN TRACEABILITY LAYER

A digital twin receives telemetry, estimates hidden state, applies analytical transformations, and exposes a controlled state to downstream optimization [33]–[35], [26].

A. IoT Telemetry and Digital-Twin Synchronization

Telemetry can include location, temperature, load, battery state, route status, authorization state, power-line-carrier events, and shipment timestamps [20], [31], [36]. Twin synchronization is declared when

$$\lim_{\{j \rightarrow \infty\}} d(\varphi_j, \varphi) = 0 \quad \text{in } \Phi \quad (21)$$

that is, every selected seminorm of the reconstructed testing-space representative stabilizes [12]. Spectral differential properties provide the operator relations required for transformed dynamics [16].

B. Green Logistics and Provenance Integrity

A route is operationally preferable when it satisfies service constraints while reducing energy use or emissions intensity [27]. A feasible green selection solves

$$\min_{\rho} \omega_E E(\rho) + \omega_G G(\rho) \quad \text{s.t.} \quad \text{service}(\rho) \geq s_{min} \quad (22)$$

Blockchain provenance is a hash chain: each record contains payload data and the digest of the previous record [28],

$$R_n = (D_n, H(R_{n-1})), \quad R_0 = (D_0, 0) \quad (23)$$

Altering an earlier transaction breaks subsequent links. The twin describes current operational state; the provenance layer supports historical trust [26], [28].

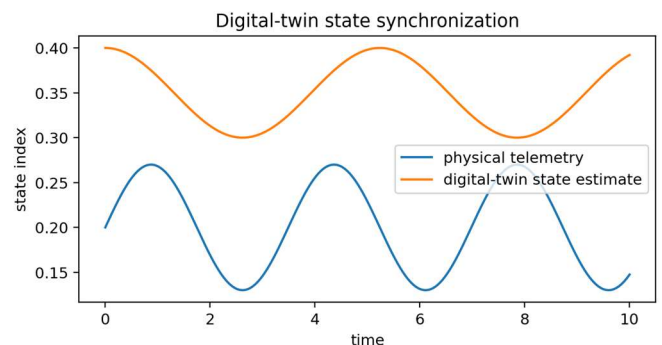


Fig. 5. Digital-twin synchronization concept linking physical telemetry with an estimated analytical state.

VII. QUANTUM-CONSTRAINED LOGISTICS, MOBILITY AND RESILIENCE

Unit Load Device configuration, vehicle assignment, route selection, charging decisions, and disruption-aware dispatch can be encoded as binary decisions. A QUBO model combines operational cost with penalty terms that enforce capacity, assignment, timing, and energy feasibility [34], [35].

A. QUBO and Quantum-Annealing Dispatch

Let x_i belong to $\{0,1\}$ indicate selection of configuration i . The QUBO energy is

$$H(x) = \sum_{i,j} x_i Q_{ij} x_j = x^T Q x \quad (24)$$

Constraints are embedded by squared penalties

$$H(x) = x^T Q_0 x + \sum_k \lambda_k (A_k x - b_k)^2 \quad (25)$$

The annealer searches the energy landscape, while a classical feasibility check interprets the returned configuration against operational constraints [8], [35]. Energy feasibility for hybrid-electric and solar-assisted packs may be written [9], [32]

$$E_{pack} \geq \frac{E_{mission}}{\eta_{use}} - E_{solar} \quad (26)$$

B. Human-Centric Control and Constrained Resilience

Optimization must not bypass authorization. Swipe-controller logic, power-line-carrier anti-theft signaling, Industry 5.0 oversight, and energy limits form explicit gates around dispatch [21], [31], [33]. Constrained resilience is the ability to recover while remaining inside admissible security, energy, service, and provenance constraints [10], [23]. A fast recovery that violates authorization or energy limits is not admissible.

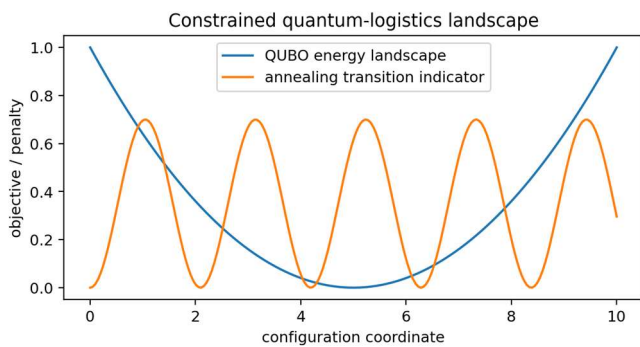


Fig. 6. Illustrative QUBO energy landscape for a constrained quantum-logistics configuration search.

The hybrid architecture is conservative: quantum annealing is a candidate-generation mechanism, whereas the digital twin, provenance layer, and human-centric authorization gates remain responsible for feasibility, trust, and accountability [20], [34].

VIII. CONCLUSION

The developed framework establishes a unified spectral foundation for representing battery states, mobility signals, multi-echelon supply-chain dynamics, disruption patterns, and digital-twin observations within a common analytical structure. The two-sided FKF component describes temporal variation, lead-time shifts, modulation, and distributional behavior, while

the FKL component represents scale-dependent inventory, capacity, demand intensity, and system-state variation.

Digital-twin states can be transformed into spectral representations in which delay, resonance, differentiation, convergence, and admissibility are examined before a recovery or routing decision is generated. Battery and solar-energy models establish physical feasibility; mobility and communication layers provide temporal and event information; blockchain-oriented traceability supplies an auditable record; and quantum annealing addresses constrained combinatorial problems such as unit-load configuration. Optimization therefore operates on an already validated system state.

Future work can extend this formulation toward experimentally validated parameter identification, multidimensional FKF–FKL operators, noisy and incomplete observations, adaptive digital-twin updating, fractional-order system models, and hardware-level validation of spectral admissibility and recovery mechanisms.

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