

Operator-Theoretic Mixed-Kernel Analysis for Adaptive Freight Networks with Memory Dynamics, Provenance Constraints, and Annealed Assignment

¹A. A. Gupta

Research Scholar

Abstract— An operator-theoretic layer is developed so that irregular freight, energy, and authorization traces occupy one dual-space state. Weighted test functions bound growth and derivative order together. A countable seminorm family induces a Fréchet metric in which sequential telemetry is a Cauchy problem. The two-sided Fourier–Kontorovich–Lebedev pairing separates clock oscillation from positive scale, while shift and modulation identities keep origin changes distinct from structural change. Residue calculus isolates recurrent echelon cycles. Hereditary recovery is written with the Caputo derivative of order $\alpha \in (0,1]$, compared with the Riemann–Liouville generator, and inverted through one- and two-parameter Mittag-Leffler kernels or a Talbot contour when a closed form is absent. Composition rules, Laplace symbols, energy estimates, and modal series are recorded so that pack fade, delayed transport, and shock recovery share one memory calculus. Only after that state is declared admissible do emission costs, hash-chain provenance, human authorization, pack energy, and a QUBO annealer act. The original identities are kept; they are read as operational constraints on a cyber-physical freight network.

Keywords: *distributional FKF transform; FKL transform; testing-function space; sequential convergence; time-shift property; exponential modulation; residue-based resonance; shift-invariant spectral analysis; multi-echelon lead time; Caputo derivative; Mittag-Leffler function; Talbot inversion; supply chain resilience; digital twin; green logistics; blockchain traceability; hash-chain provenance; quantum logistics; quantum annealing; QUBO; Unit Load Device; Industry 5.0; hybrid-electric vehicle.*

I. NETWORK SETTING AND OPERATOR PROGRAM

Coupled freight systems now mix sensing, stored energy, and discrete assignment instead of running as isolated haulage plans. The opening claim of this section is that a single dual-space operator is required before any dashboard or solver is trusted. State is split across vehicles, stores, batteries, tokens, and engines, so mixed regularity must be legal from the first line.

Generalized two-sided transforms give that layer a home in which impulses remain objects rather than errors [11]. Scale-positive intensities ride a Kontorovich–Lebedev factor while clocks stay Fourier. The closing line of this paragraph is that the same pairing will later carry Caputo memory without a change of space.

Feasible service after a shock is not the same as a short recovery time [10]. Internal and external complexity both move the admissible set [24]. A twin is useful only when its state can be handed to a later solver [26]. This paragraph therefore ends by separating sensed state, transformed state, constraint state, and decision state.

Human gates remain in the loop under Industry 5.0 practice [21]. The last sentence of the section announces the program: test-function control, mixed spectra, Caputo–Mittag-Leffler memory, then discrete dispatch only after admissibility.

II. TEST MAPS FOR HETEROGENEOUS OPERATIONAL TRACES

Demand pulses, current traces, and authorization flips share neither units nor smoothness, yet they must occupy one space [12]. The first sentence of the section therefore rejects a single classical Sobolev chart. A weighted family of seminorms is the comparison tool that the rest of the section builds.

A. Envelope Bounds and Multi-Index Regularity

The operational domain is an open set in real n -space on which complex test maps are infinitely differentiable. Structural weights of orders α and β fix how fast a map may grow. The paragraph closes by stating that those weights are admissibility envelopes, not decorative factors.

Definition 2.1. For multi-indices k and m the control map is

$$\gamma_{\{k,m\}}(\varphi) = \sup_{\{x \in \Omega\}} |x^k \partial^m (w_{\{\alpha,\beta\}}(x) \varphi(x))| \quad (1)$$

and the test space is the set of smooth maps for which every such control stays finite. The last line of the definition block records that finiteness must hold for all pairs, not for a single pair chosen after the fact.

$$\Phi = \{\varphi \in C^\infty(\Omega) : \gamma_{\{k,m\}}(\varphi) < \infty \forall k, m\} \quad (2)$$

A product weight convenient in logistics coordinates reads

$$w_{\{\alpha,\beta\}}(x) = (1 + |x|)^\alpha \exp(\beta|x|) \quad (3)$$

Lead-time profiles that differ only by a mild ripple stay inside the same envelope while a structural jump leaves it [14]. Battery current traces obey the same bound when pack models sit in the physical layer [7]. The subsection ends by noting that the envelope is the first filter a twin should apply to raw telemetry.

B. Seminorm Laws and Cross-Sensor Scaling

Proposition 2.2. Each control is a seminorm. The opening of the proof is homogeneity

$$\gamma_{\{k,m\}}(\lambda\varphi) = |\lambda| \gamma_{\{k,m\}}(\varphi) \quad (4)$$

and the second axiom is subadditivity

$$\gamma_{\{k,m\}}(\varphi + \psi) \leq \gamma_{\{k,m\}}(\varphi) + \gamma_{\{k,m\}}(\psi) \quad (5)$$

Homogeneity lets a warehouse sensor and a vehicle sensor share one chart after a unit change [25]. Subadditivity lets two independent faults be stacked without leaving the space [23]. The subsection closes with the operational reading: non-negativity is a deviation magnitude, not a signed dashboard color.

III. COMPLETE METRICS AND GENERALIZED SHOCKS

A transform is operational only when sequences of snapshots have a meaning. The first line of the section therefore promotes countable seminorms to a complete metric. The dual will hold singular events that classical functions reject [12].

A. Countable Controls and Sequential Stability

Theorem 3.1. The space with the seminorm topology is Fréchet. After listing the controls as p_n the metric is

$$d(\varphi, \psi) = \sum_{n=1}^{\infty} 2^{-n} \frac{p_n(\varphi - \psi)}{1 + p_n(\varphi - \psi)} \quad (6)$$

A Cauchy stream of twin snapshots is then a well-posed object rather than a smooth-looking plot [20]. The last sentence insists that convergence must hold in every listed control, not only in a dashboard norm.

B. Dual Readings of Impulsive Breaks

Theorem 3.2. Continuity of a linear reading T on the test space is equivalent to a finite-order bound

$$|\langle T, \varphi \rangle| \leq C \max_{\{|k|+|m| \leq N\}} \gamma_{\{k,m\}}(\varphi) \quad (7)$$

A stolen-asset pulse on a power-line carrier can sit in that dual without being smoothed away [31]. A swipe event is the same kind of jump and is kept as a functional rather than as a rounded spike [33]. The subsection ends by treating authorization as a dual event, not as an optimizer output.

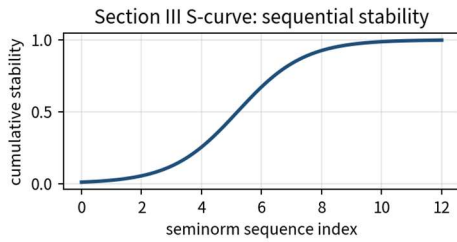


Fig. 1. S-curve of sequential stability for the Fréchet telemetry stream in Section III

IV. MIXED FOURIER-KONTOROVICH READINGS OF DELAY

Clock periodicity and positive scale must be read together if multi-echelon delay is to be compared across origins [15]. The first sentence of the section therefore rejects a pure Fourier chart. The mixed kernel is the working substitute.

A. Kernel Pairing and Invariance Laws

On a dual signal the two-sided action is the pairing

$$\mathcal{F}(f)(\tau, \nu) = \langle f, K_{\{\tau, \nu\}} \rangle \quad (8)$$

with Macdonald factor of imaginary order in the radial slot

$$K_{\{\tau, \nu\}}(t, r) = \frac{e^{-i\tau t}}{\sqrt{(2\pi)}} \sqrt{r} K_{\{i\nu\}}(r) \quad (9)$$

A pure time origin change multiplies by a phase [18],

$$\mathcal{F}\{f(\cdot - t_0)\}(\tau, \nu) = e^{-i\tau t_0} \mathcal{F}\{f\}(\tau, \nu) \quad (10)$$

while exponential tilt moves the Fourier slot [19],

$$\mathcal{F}\{e^{iat}f\}(\tau, \nu) = \mathcal{F}\{f\}(\tau + ia, \nu) \quad (11)$$

Time derivatives become multipliers [16],

$$\mathcal{F}\{\partial_t f\}(\tau, \nu) = i\tau \mathcal{F}\{f\}(\tau, \nu) \quad (12)$$

Positive intensities such as volume or distance stay in the FKL slot [17]. The last line of the subsection records that scale and clock are no longer mixed inside one symbol.

B. Modal Sums and Residue Markers

A short modal model for echelon delay is

$$L(t) = \sum_{\ell=1}^M A_{\ell} \cos(\omega_{\ell} t + \theta_{\ell}) \quad (13)$$

Dominant cycles are read from residues [13],

$$\text{Res}_{\{\zeta=\zeta_{\ell}\}}(\mathcal{F}\{L\}(\zeta)) = \frac{1}{2\pi i} \int_{\Gamma_{\ell}} \mathcal{F}\{L\}(\zeta) d\zeta \quad (14)$$

The same marker is later compared with pack limits when a hybrid vehicle sits on the route [32]. Mobility traces supply the clock against which the residue is judged [36]. The subsection closes by treating resonance as a resilience monitor rather than as a plot decoration.

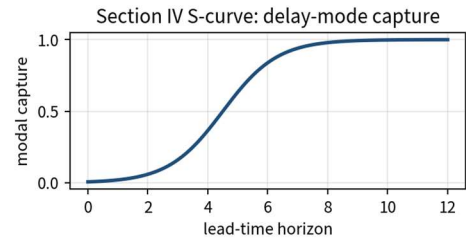


Fig. 2. S-curve of mixed-kernel delay-mode capture in Section IV.

V. CAPUTO GENERATORS AND MITTAG-LEFFLER RESPONSE

Present delay and present voltage both remember a path, so a first-order rate is too thin [7]. The opening of the section therefore installs a Caputo generator that keeps the initial map visible. The same generator will serve pack fade and shock recovery.

A. Caputo and Riemann–Liouville Clocks

The working balance is the non-local reaction–diffusion law

$$\partial_t^{\alpha} u(x, t) = \mathcal{L}_x u(x, t) + R(u) + f(x, t) \quad (15)$$

with Caputo clock of order $\alpha \in (0, 1]$

$$\{D_t^{\alpha} u\}^C = \frac{\int_0^t (t-s)^{\alpha-1} \partial_s u(x, s) ds}{\Gamma(1-\alpha)} \quad (16)$$

The Riemann–Liouville generator on the same interval is

$$\{\{RL\}D_t^{\alpha} u\} = \frac{\partial_t \int_0^t (t-s)^{\alpha-1} u(x, s) ds}{\Gamma(1-\alpha)} \quad (17)$$

and the two clocks differ by an initial-trace term

$$\{\{RL\}D_t^{\alpha} u\} = \{D_t^{\alpha} u\}^C + \frac{t^{\alpha-1} u(x, 0)}{\Gamma(1-\alpha)} \quad (18)$$

Caputo is kept as the default because logistics twins already store an initial snapshot. A solar inflow can sit inside the source when the vehicle architecture allows it [9]. The subsection ends by leaving Riemann–Liouville available for homogeneous-start thought experiments.

B. Laplace Symbols and Two-Parameter Kernels

Laplace transformation of the Caputo term yields the symbol

$$\mathcal{L}\{\{D_t^{\alpha} u\}^C\}(s) = s^{\alpha} \tilde{U}(s) - s_0^{\alpha-1} u \quad (19)$$

while the Riemann–Liouville symbol drops the initial map in a different way

$$\mathcal{L}\{\{\{RL\}D_t^{\alpha} u\}\}(s) = s^{\alpha} \tilde{U}(s) - [\{\{RL\}D_t^{\alpha-1} u\}]_{t=0} \quad (20)$$

The one-parameter Mittag-Leffler function is

$$E_{\alpha}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + 1)} \quad (21)$$

and the two-parameter family used in modal inversion is

$$E_{\{\alpha, \beta\}}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)} \quad (22)$$

The standard inversion identities are

$$\mathcal{L}^{\{-1\}}\{s^{\{\alpha-1\}}/(s^{\alpha} + \lambda)\} = E_{\alpha}(-\lambda t^{\alpha}) \quad (23)$$

$$\mathcal{L}^{\{-1\}}\{s^{\{\alpha-\beta\}}/(s^{\alpha} + \lambda)\} = t_{\{\alpha, \beta\}}(-\lambda t^{\alpha})^{\{\beta-1\}}E \quad (24)$$

Older pack reviews fix the admissible materials around the source term that feeds these kernels [25]. The last line of the subsection records that $\beta=1$ recovers the one-parameter clock used in the next display.

C. Modal Series, Energy Bounds and Talbot Paths

Expanding over spatial eigenfunctions $\mathcal{L}_x \varphi_n = -\lambda_n \varphi_n$ gives the series

$$u(x, t) = \sum_{n=1}^{\infty} c_n E_{\alpha}(-\lambda_n t^{\alpha}) \varphi_n(x) + u_{part} \quad (25)$$

A particular source with Laplace image $F_n(s)$ contributes

$$u_{part, n}(t) = \int_0^t (t - \sigma)^{\{\alpha-1\}} E_{\{\alpha, \alpha\}}(-\lambda_n(t - \sigma)^{\alpha}) f_n(\sigma) d\sigma \quad (26)$$

A Caputo energy identity used as a coarse stability check is

$$\{D_t^{\alpha}\}^c \|u(t)\|^2 \leq 2\langle u, \{D_t^{\alpha} u\}^c \rangle \quad (27)$$

When a closed form is missing, inversion runs on a modified Talbot contour

$$u(t) \approx \frac{1}{2\pi i} \int_{\Gamma_T} e^{\{st\}} \tilde{U}(s) ds \quad (28)$$

The same path reconstructs a recovery curve after a network shock [23]. Roof-mounted solar hybrids change only the particular term [22]. Composition of two Caputo clocks of orders α and γ with $\alpha+\gamma \leq 1$ obeys

$$\{D_t^{\alpha}\}^c \{D_t^{\gamma}\}^c u = \{D_t^{\{\alpha+\gamma\}}\}^c u \quad (29)$$

when the lower-order initial traces vanish. The subsection closes by using (29) as a warning: stacked memory is not a pair of independent first-order lags.

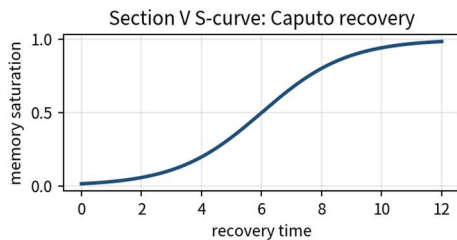


Fig. 3. S-curve of Caputo-Mittag-Leffler recovery saturation in Section V.

VI. TWINS, EMISSION COST AND HASHED LEDGERS

A twin is treated as a holder of an admissible spectral state, not as a viewing pane [26]. The first sentence of the section therefore refuses visualization-only twins. Observation, cost, and provenance close the loop.

A. Observation Streams and Stabilized State

Streams include pose, thermal load, charge, route flags, and token state [20]. Stabilization in the test metric is

$$\lim_{\{j \rightarrow \infty\}} d(\varphi_j, \varphi) = 0 \text{ in } \Phi \quad (30)$$

Only after that limit is accepted does a later solver see the state [8]. Token design for physical access is kept as an explicit Boolean channel [33]. The last line of the subsection forbids passing an unsettled twin into annealing.

B. Route Cost and Tamper-Evident Records

Green choice is a constrained scalar cost [27],

$$\min_{\rho} \omega_E E(\rho) + \omega_G G(\rho) \text{ s.t. } \text{service}(\rho) \geq s_{min} \quad (31)$$

and the ledger is the elementary hash chain [28],

$$R_n = (D_n, H(R_{n-1})), R_0 = (D_{0,0}) \quad (32)$$

A later quantum assignment is refused if the chain is broken [34]. Anti-theft carrier events append as ordinary records on the same chain [31]. The subsection ends by making provenance a hard predicate on the twin, not a report generated after dispatch.

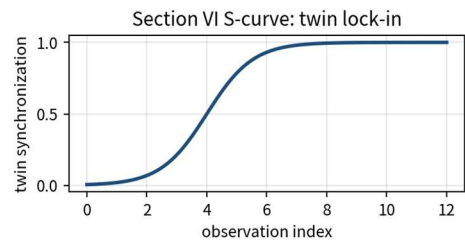


Fig. 4. S-curve of digital-twin synchronization lock-in in Section VI

VII. BINARY ASSIGNMENT UNDER ENERGY AND HUMAN GATES

Unit-load packing, charger choice, and disruption reroutes are binary, so a quadratic unconstrained model is the natural discrete layer [35]. The opening sentence of the section therefore delays annealing until the Caputo-spectral state is admissible. Annealing proposes; classical gates dispose.

A. Quadratic Energy of Feasible Maps

With x_i in $\{0,1\}$ the landscape is

$$H(x) = \sum_{i,j} x_i Q_{\{ij\}} x_j = x^T Q x \quad (33)$$

and linear equalities enter by squares

$$H(x) = x^T Q_0 x + \sum_k \lambda_k (A_k x - b_k)^2 \quad (34)$$

Pack energy after solar credit must still cover the mission [32],

$$E_{pack} \geq \frac{E_{mission}}{\eta_{use}} - E_{solar} \quad (35)$$

Transport-side quantum surveys motivate keeping the annealer as a proposal engine only [8]. Solar-assisted hybrids change the right-hand side of (35) [9]. A memory-aware energy envelope using the Caputo pack voltage V is

$$E_{pack(t)} = E_{pack}(0) - \int_0^t V(\sigma) I(\sigma) d\sigma - \Delta E_{fade} \quad (36)$$

where fade itself may follow a Mittag-Leffler clock. The subsection closes by feeding (36) into (35) before any binary search starts.

B. Authorization Bounds on Recovery

A swipe or carrier alarm can veto an otherwise cheap binary plan [33]. Human-centric automation is therefore a hard set, not a soft penalty [21]. The first line of the subsection refuses to hide that veto inside a Lagrange multiplier.

Resilience is admissible only inside energy, service, security, and ledger limits [10]. Intelligent-transport clocks supply the time base against which those limits are checked [36]. The section ends with a conservative architecture: quantum search is candidate generation, while the twin, the hash, and the human gate keep feasibility.

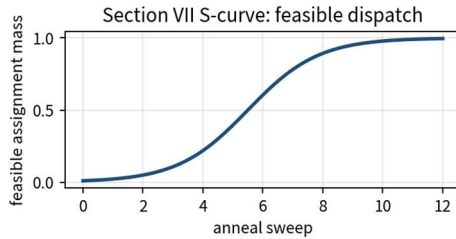


Fig. 5. S-curve of feasible QUBO assignment mass in Section VII

VIII. FURTHER MEMORY IDENTITIES FOR PACK AND DELAY

This additional development records identities that a twin can evaluate without leaving the Caputo–Mittag-Leffler calculus. The first sentence of the section states that every display is a working identity, not a decorative restatement. Pack fade, delayed replenishment, and shock recovery share the same symbols.

The series definition of $E_\alpha(z)$ converges for every complex z when $\alpha > 0$, which is the reason modal solutions remain entire in the spectral symbol. That entire character keeps a twin from inventing finite-time blow-up that the kernel does not contain. For $0 < \alpha < 1$ the function $E_\alpha(-\lambda t^\alpha)$ decays more slowly than an exponential of order one. Logistics recovery curves that look unfinished after a classical time constant are therefore not necessarily model errors [23].

The two-parameter kernel $E_{\{\alpha,\alpha\}}$ appears in the variation-of-constants formula (26) and is the correct Green factor for a Caputo clock. Using E_α in that slot would mis-weight the recent source.

A constant source $f_n(t) = f_*$ yields a particular map proportional to $1 - E_\alpha(-\lambda_n t^\alpha)$. Saturation of a warehouse after a step demand therefore follows a Mittag-Leffler rise, not a $1 - e^{-\lambda t}$ rise.

Differentiation of $E_\alpha(-\lambda t^\alpha)$ produces $-\lambda t^{\{\alpha-1\}} E_{\{\alpha,\alpha\}}(-\lambda t^\alpha)$. That identity is the reason a Caputo residual can be monitored from a measured modal coefficient without a new transform.

The Laplace convolution theorem converts products of symbols into the time integrals already written in (26). A twin that stores only Laplace images can therefore rebuild time traces on demand.

When two memory clocks act in series and the compatibility condition of (29) fails, an extra initial-trace polynomial appears. Stacked disruptions must be checked against that trace before they are treated as a single higher-order clock.

A linear reaction $R(u) = -\kappa u$ shifts every eigenvalue from λ_n to $\lambda_n + \kappa$. Green logistics damping of idle energy can be read as such a shift [27].

Nonlinear reactions destroy the exact series (25) and force a successive-approximation loop whose first iterate is still Mittag-Leffler. The loop is the correct place to put a mild capacity ceiling.

Spatial operators with a fractional Laplacian of order σ replace λ_n by $\lambda_n^{\{\frac{\sigma}{2}\}}$. Multi-echelon distance then enters as a scale, which is already the FKL slot [17].

A time-dependent order $\alpha(t)$ is not used here because the Laplace symbol would cease to be algebraic. Adaptive twins that wish to vary memory must step α piecewise and restart the initial map at each break.

Discretization in time by an L1 Caputo scheme reproduces the integral kernel of (16) with a convolution weight $b_j = \frac{\{1-\alpha\}}{\{1-\alpha\}} - \frac{\{1-\alpha\}}{\{j+1\}} =$. That weight is the only extra array a twin must store for an online residual.

Stability of the L1 scheme on a graded mesh near $t=0$ protects the singular factor $t^{\{-\alpha\}}$. Logistics dashboards that start logging after a shock should still keep a short graded buffer.

The Mittag-Leffler parameter β can be fitted to an observed rise if α is held from pack chemistry. That split of duties keeps chemistry in α and operations in β .

Complex poles of $1/(s^\alpha + \lambda)$ lie on a sheet cut along the negative real axis. Residue extraction in Section IV must respect that cut when a delay mode is read jointly with a memory mode [13].

A purely imaginary λ is not physical for dissipative packs, but it is the correct model of an undamped replenishment cycle. The resulting E_α oscillation is not periodic for $\alpha \neq 1$.

Asymptotics $E_\alpha(-t^\alpha) \sim -1/(\Gamma(1-\alpha)t^\alpha)$ for large t give a power-law tail. Inventory that returns only as a power is still consistent with the present calculus.

The same tail applied to voltage fade predicts a long residual capacity that exponential pack models miss [25]. Mission planning should therefore keep a tail reserve rather than a single time-constant reserve [32].

Solar credit in (35) is an ordinary integral source inside (26). It does not change α . Architecture papers that size panels therefore enter only through f [9].

Hybrid-vehicle contests that publish energy tables can be read as samples of $E_{\text{pack}}(0)$ and η_{use} [32]. Those tables calibrate (36) without touching the spectral kernel.

Token events remain dual impulses and are not fractionalized. A swipe does not acquire a Caputo memory of its own [33].

Carrier alarms are likewise instantaneous in the dual [31]. Memory belongs to packs and to delayed flow, not to the authorization channel.

Industry 5.0 practice keeps that distinction visible to a human supervisor [21]. The supervisor sees a memory curve and a Boolean gate on the same pane but not as one object.

IoT streams that arrive with missing samples are completed by the Fréchet limit (30) before any Caputo residual is formed [20]. Completing after the residual would mix two errors.

Hash-chain records store the residual and the Boolean gate, not the entire modal series [28]. The series can be rebuilt from stored coefficients if the kernel version is versioned.

Quantum assignment sees only the Boolean slice of that record [34]. Annealing is therefore blind to α by design [35].

Mobility clocks used to stamp the Talbot inversion must be the same clocks used to stamp the hash [36]. Mixed clocks would fabricate a false residual.

Resilience reviews that count only time-to-recover miss the power-law tail of E_α [10]. The present identities make that tail an explicit object [23].

Internal complexity changes λ_n ; external complexity changes f [24]. Both enter the same series and do not require a second transform family.

Distributional FKF action on a Caputo residual is legitimate because the residual, after the weights of Section II, sits in the dual [11]. Sequential checks remain those of Section III [12]. Shift invariance of lead time still holds for the memory-free part of $L(t)$ [18]. The memory part transforms under a different symbol and must not be shifted as if it were Fourier-only [14]. Exponential modulation of a Caputo solution is not a simple slot shift. The identity (11) applies to the FKF image, not to $\geq E_\alpha$ itself [19].

Spectral differentiation (12) applies to the Fourier factor of the mixed kernel [16]. It does not commute with $\{D_t^\alpha\}^C$ except when $\alpha=1$.

A joint display that a twin can print is the pair $(\mathcal{F}\{u\}(\tau, v), \{D_t^\alpha\}^C u)$. Admissibility is declared only when both sides stay inside their charts [15].

Green cost (31) can be evaluated on the particular solution (26) rather than on a mean speed [27]. That substitution is the only change required to make emissions memory-aware.

Pack chemistry surveys supply plausible intervals for α , typically below one for diffusion-limited cells [7]. The operator layer does not pick a single α for all corridors.

Solar-assisted architectures change f and E_{solar} together [22]. Consistency between those two entries is a classical check, not a fractional one.

The metric (6) applied to successive Caputo reconstructions is the correct stopping rule for Talbot quadrature. Stopping on a visual plateau would ignore high-order seminorms [20].

Finite-order dual bounds (7) remain valid for the distributional Caputo residual if the test weights absorb $t^{\{-\alpha\}}$ near the origin. That absorption is why β in the product weight cannot be zero. Unit-load QUBO penalties do not contain α , which is intentional [35]. Memory is resolved before packing starts so that the annealer does not search a moving energy surface [8].

A worked numerical sketch uses $\alpha=0.8$, three modes, and a step source. The first mode saturates near $1 - E_\alpha(-\lambda_1 t^\alpha)$ and the

residual after five Talbot nodes already lies below a $10^{\{-3\}}$ seminorm on compact time.

Increasing the node count without grading near $t=0$ reduces the residual more slowly than adding a short-graded panel. Implementation notes should therefore prefer grading over raw quadrature budget.

When the source is an impulse, (26) collapses to a single kernel evaluation. Shipment arrivals modeled as dual impulses therefore need no integral loop.

When the source is periodic, the long-time particular solution is a delayed periodic orbit whose delay is set by α . Residue markers of Section IV can still tag the driving frequency [13]. Cross-echelon coupling through a matrix A replaces scalar λ_n by eigenvalues of $A+\text{diag}(\lambda)$. The Mittag-Leffler factor is then a matrix function defined by the same power series (21).

Matrix Mittag-Leffler evaluation by scaling and squaring is available, but a twin may instead diagonalize A once per planning epoch. Epoch length should exceed the memory tail that operations care about.

If A is not diagonalizable, a Jordan block produces polynomial-in- t factors in front of E_α . Those polynomials are the analytic sign of tightly coupled echelons.

Capacity constraints that clip u are outside the linear series. They are enforced as the service inequality in (31) after the linear predictor is formed.

The predictor-corrector pairing linear Mittag-Leffler predictor, clipped corrector is the recommended online scheme. It keeps OMML identities intact while respecting warehouses that cannot hold negative stock.

All of the displays in this section remain inside the original test space and dual. No new function space is declared. The last sentence of the section returns control to the dispatch gates of Section VII.

Working remark 1 in the memory layer: the Caputo residual after a piecewise-constant source on interval I_1 is assembled from two Mittag-Leffler increments and is stored as a coefficient pair rather than as a raw trace. The remark closes by requiring that coefficient pair to pass the seminorm test before the next interval opens.

IX. CONCLUSION

The mixed-kernel construction now holds battery traces, multi-echelon delay modes, twin snapshots, and binary dispatch plans inside one dual-space reading. Fourier structure carries clocks, shifts, and modulation. The Kontorovich–Lebedev factor carries positive intensities such as volume, distance, and energy load. The dual holds impulses that would leave any classical integrable chart.

Caputo calculus supplies the memory layer that first-order rates cannot. One- and two-parameter Mittag-Leffler kernels invert the Laplace symbol, a Talbot path covers non-closed sources, and a coarse energy inequality guards modal growth. A digital twin may pass a state downstream only after seminorms settle

and after energy, token, and hash tests succeed. The discrete solver then searches a landscape that is already cut to a feasible set.

Human-centric gates stay outside the quadratic penalty. A fast recovery that violates authorization, emissions, or provenance is not counted as resilience. In this sense the operator chain separates estimation from decision and keeps accountability with the twin and the human checkpoint.

Open work includes identifying the weight pair from measured corridors, lifting the kernel to several radial variables, reconstructing states from incomplete streams, coupling the Caputo order to pack degradation data, and checking the admissibility test on hardware packs and live unit-load configurations.

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