Pitch Attitude Stabilization Control for the Ultra Stick 25E Unmanned Aerial Vehicle Under Disturbances

Tran Duc Dai[#1], Dao Hung Cuong[#2], Nguyen Thanh Lam[#3]
[#]*Master of Control & Automation, Lecturer, Faculty of Aviation Equipment,*
Air Force Officer College, Khanh Hoa Province, Vietnam
[1]tranducdai2610@gmail.com, [2]hungcuongpkkq@gmail.com, [3]nguyenthanhnam.nam89@gmail.com

Abstract:
This paper presents the design of a pitch attitude stabilization control system for the Ultra Stick 25E unmanned aerial vehicle (UAV) operating under external disturbances. Based on a longitudinal flight dynamics model linearized around an equilibrium trim condition, a Linear Quadratic Gaussian (LQG) optimal control strategy is synthesized by integrating a Linear Quadratic Regulator (LQR) with an optimal Kalman filter. Numerical simulations in MATLAB/Simulink demonstrate that the proposed LQG controller significantly enhances transient performance—reducing the settling time from 17.5 s to 2.16 s and the overshoot from 22.5 percent to 4.1 percent—while effectively attenuating both process and measurement noise. Consequently, the LQG architecture ensures superior pitch tracking accuracy and robustness compared to conventional LQR control.

Keywords—Ultra Stick 25E UAV; pitch channel; LQR/LQG optimal control; Kalman filter; aerodynamic disturbances.

I. INTRODUCTION

In practical unmanned aerial vehicle (UAV) operations, maintaining attitude stability against external disturbances—such as atmospheric turbulence, wind gusts, and sensor noise—remains a critical technical challenge [3]. Particularly for lightweight fixed-wing UAVs such as the Ultra Stick 25E, disturbance-induced pitch oscillations not only degrade flight trajectory tracking accuracy but also accelerate mechanical fatigue and wear in elevator actuators.

Although UAVs increasingly execute complex civil and military missions, conventional flight controllers often exhibit limited precision due to unoptimized energy–error cost functions or suffer severe performance degradation when feedback signals are corrupted by sensor noise. To address these limitations, this study applies Linear Quadratic Gaussian (LQG) optimal control [1], [2] to stabilize the pitch attitude of the Ultra Stick 25E UAV under simultaneous process and measurement disturbances. Specifically, an optimal Kalman filter [1] is employed to estimate dynamic state vectors corrupted by noise, providing accurate state feedback for the Linear Quadratic Regulator (LQR) [2] to execute optimal control laws and ensure robust closed-loop stability. The remainder of this paper is organized as follows: Section II formulates the longitudinal mathematical model of the Ultra Stick 25E UAV; Section III details the synthesis of the LQR and LQG controllers; Section IV evaluates the MATLAB/Simulink simulation results; and Section V concludes the paper.

II. LONGITUDINAL DYNAMICS MODEL OF THE ULTRA STICK 25E UAV

To formulate the spatial equations of motion for the UAV, a body-fixed coordinate frame *Oxyz* attached to the vehicle's center of gravity is defined, as illustrated in Fig. 1.

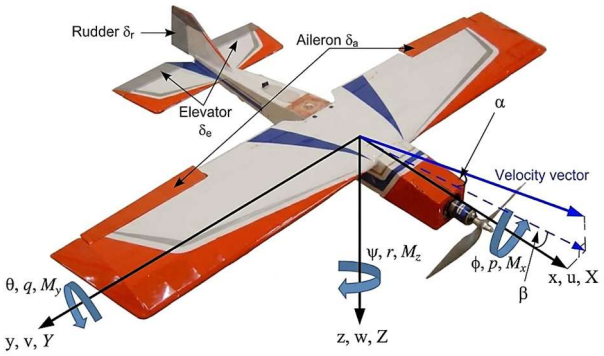

Fig. 1. Coordinate frames and motion channels of the UAV.

Grounded in rigid-body mechanics and aerodynamics [7], the six-degree-of-freedom (6-DOF) equations of motion for a fixed-wing UAV are derived via the Newton–Euler equations [5]. The spatial motion of the UAV is decomposed into two components [7]: translational motion of the center of mass and rotational motion about the center of mass as an ideal rigid body:

$$\begin{cases} m\dfrac{d\vec{W}}{dt} = \sum \vec{F} \\ \dfrac{d\vec{K}}{dt} = \sum \vec{M} \end{cases} \tag{1}$$

where *W* denotes the linear velocity vector and *K* represents the angular momentum vector of the UAV.

Projecting (1) onto the body-fixed coordinate frame *Oxyz* yields the nonlinear differential equations governing the general spatial motion of the UAV:

$$\begin{cases} m(\dot{W}_x + \omega_y W_z - \omega_z W_y) = \sum F_x \\ m(\dot{W}_y + \omega_z W_x - \omega_x W_z) = \sum F_y \\ m(\dot{W}_z + \omega_x W_y - \omega_y W_x) = \sum F_z \\ J_x\dot{\omega}_x + (J_z - J_y)\omega_y\omega_z = \sum M_x \\ J_y\dot{\omega}_y + (J_x - J_z)\omega_x\omega_z = \sum M_y \\ J_z\dot{\omega}_z + (J_y - J_x)\omega_x\omega_y = \sum M_z \end{cases} \tag{2}$$

Pitching motion occurs when the UAV maneuvers within the vertical plane of symmetry *Oxz*, commonly referred to as longitudinal motion. By decoupling the longitudinal dynamics (assuming lateral-directional state variables are zero), the governing differential equations reduce to:

$$\begin{cases} m(\dot{W}_x + \omega_y W_z - \omega_z W_y) = \sum F_x \\ m(\dot{W}_y + \omega_z W_x - \omega_x W_z) = \sum F_y \\ J_z \dot{\omega}_z + (J_y - J_x)\omega_x \omega_y = \sum M_z \end{cases} \tag{3}$$



Fig. 2. Fixed-wing Ultra Stick 25E UAV.

According to experimental parameters from the University of Minnesota [6], the baseline physical characteristics of the Ultra Stick 25E UAV are as follows: mass m = 1.959 kg, wingspan b = 1.27 m, wing area S = 0.31 m^2, mean aerodynamic chord c = 0.25 m, and Oswald efficiency factor e = 0.75.

Applying a first-order Taylor series expansion around the equilibrium trim condition linearizes the longitudinal differential equations in (3) into the following state-space representation:

$$\dot{x}_{\text{lon}} = Ax_{\text{lon}} + B\delta_e$$

$$\dot{x}_{\text{lon}} = \begin{bmatrix} X_u & X_w & -X_q - W_e & -g\cos\theta_e \\ Z_u & Z_w & Z_q + U_e & -g\sin\theta_e \\ M_u & M_w & M_q & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} X_{\delta_e} \\ Z_{\delta_e} \\ M_{\delta_e} \\ 0 \end{bmatrix} \delta_e \tag{4}$$

where the longitudinal state vector is $x_{\text{lon}} = [u,\ w,\ q,\ \theta]^T$, comprising the axial velocity perturbation u, normal velocity perturbation w, pitch rate q, and pitch angle θ; $X(.)$, $Z(.)$, and $M(.)$ denote the longitudinal aerodynamic force and pitching moment stability derivatives evaluated at the trim point; δ_e represents the elevator deflection control input; and U_e, W_e, and θ_e denote the steady-state velocity components and pitch angle at equilibrium flight, respectively.

Substituting the empirical parameters of the Ultra Stick 25E UAV [6] into (4) yields the explicit longitudinal state-space model:

$$\begin{bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.38 & 0.60 & -0.36 & -9.8 \\ -0.98 & -7.81 & 15.32 & -0.21 \\ 0.18 & -8.31 & -35.2 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} -0.36 \\ -3.62 \\ -106.32 \\ 0 \end{bmatrix} \delta_e \tag{5}$$

The controlled output of the system is the pitch angle $y(t) = \theta(t) = C\,x_{\text{lon}}(t)$, with the output matrix defined as C = [0 0 0 1].

III. Control System Synthesis Under Disturbances

A. Optimal LQR State-Feedback Controller Design

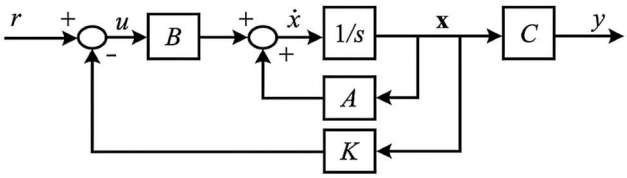

Fig. 3. Block diagram of the LQR control system.

To optimize the closed-loop transient response, the LQR control law determines the state-feedback control signal $u(t) = -K\,x(t)$ that minimizes the quadratic cost function J [2]:

$$J = \frac{1}{2}\int_0^{\infty} \left[x^T(t)Qx(t) + u^T(t)Ru(t)\right] dt \tag{6}$$

where $Q = Q^T \geq 0$ is the positive-semidefinite state weighting matrix and $R = R^T > 0$ is the positive-definite control weighting matrix. The optimal feedback gain matrix $K = R^{-1}B^TP$ is obtained from the unique positive-definite solution P of the continuous-time algebraic Riccati equation (CARE) [2]:

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \tag{7}$$

Selecting the state weighting matrix Q = diag(0.5, 0.1, 0, 1):

$$Q = \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and control weight R = 1, solving the algebraic Riccati equation in (7) in MATLAB yields the optimal LQR state-feedback gain vector K:

$$K = \begin{bmatrix} 0.5340 & -0.0133 & -0.0949 & -3.4452 \end{bmatrix} \tag{8}$$

However, an inherent limitation of the LQR architecture is its requirement for direct, noise-free measurement of the complete state vector $x(t)$—an assumption rarely satisfied in practical flight environments subject to sensor noise [1].

B. Disturbance-Rejecting LQG Controller Architecture

In realistic flight environments, the UAV state-space dynamics are inevitably subjected to process noise (aerodynamic disturbances) $w(t)$ and measurement noise (sensor inaccuracies) $v(t)$:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Gw(t) \\ y(t) = Cx(t) + v(t) \end{cases} \tag{9}$$

Assume that $w(t)$ and $v(t)$ are uncorrelated zero-mean Gaussian white noise processes with covariance matrices $E[ww^T] = Q_N \geq 0$ and $E[vv^T] = R_N > 0$, respectively.

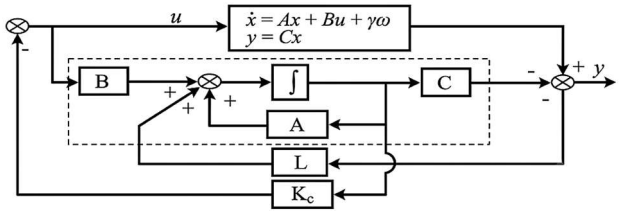

Fig. 4. Block diagram of the LQG controller architecture.

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By the separation principle, the LQG controller is synthesized by coupling an optimal Kalman filter with the LQR state-feedback regulator. Acting as an optimal stochastic state observer, the Kalman filter continuously reconstructs the estimated state vector $\hat{x}(t)$ from the control input $u(t)$ and noisy output measurements $y(t)$ according to the observer dynamics [1], [2]:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L[y(t) - C\hat{x}(t)] \tag{10}$$

where the Kalman filter gain matrix L is computed to minimize the steady-state estimation error covariance via the filter algebraic Riccati equation:

$$L = 10^{-3} \cdot \begin{bmatrix} 0.5078 \\ -0.0396 \\ 0.0119 \\ 0.0225 \end{bmatrix} \tag{11}$$

Consequently, the estimated state-feedback control law for the LQG controller is defined by $u(t) = -K\,\hat{x}(t)$ [1].

IV. Simulation Results and Performance Evaluation

Using MATLAB/Simulink [4], the pitch attitude control system of the Ultra Stick 25E UAV is evaluated under two scenarios—nominal (noise-free) and disturbed (noisy)—to comprehensively assess transient dynamics and disturbance rejection performance.

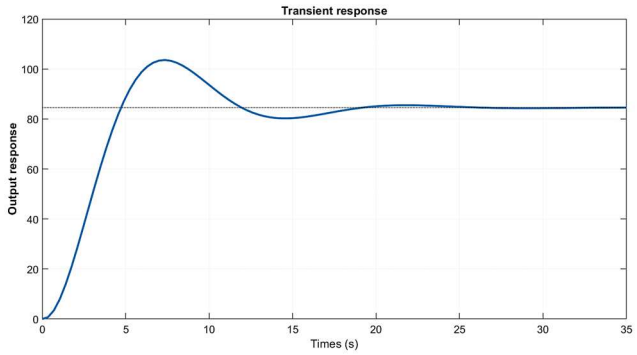

Fig. 5. Open-loop step response of the uncompensated system.

Fig. 5 illustrates the unit-step response of the uncompensated open-loop system. Although inherently stable, the baseline plant exhibits a sluggish settling time (t_s = 17.5 s), high overshoot (σ% = 22.5%), and significant steady-state error, necessitating an optimal closed-loop controller to improve dynamic performance.

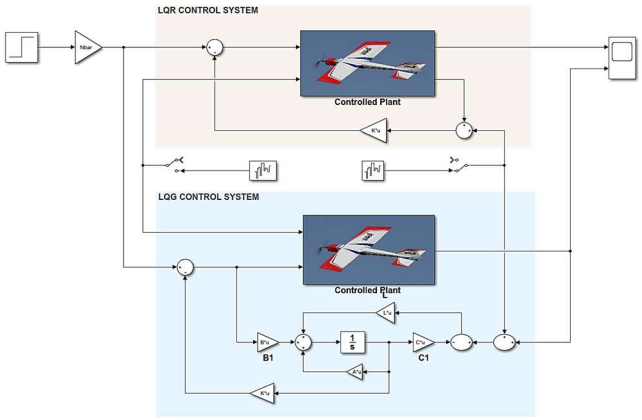

Fig. 6. Simulink simulation model of the LQR and LQG control systems for the UAV pitch channel.

The transient response performance metrics comparing the uncompensated system against the closed-loop LQR and LQG control systems are summarized in Table I.

TABLE I. Comparison of Transient Response Performance Metrics

Control Architecture	Rise Time	Settling Time	Overshoot
Uncompensated System	3.14 s	17.5 s	22.5%
LQR Controller	0.79 s	2.16 s	4.1%
LQG Controller	0.79 s	2.16 s	4.1%

A. Nominal Performance (Noise-Free Scenario)

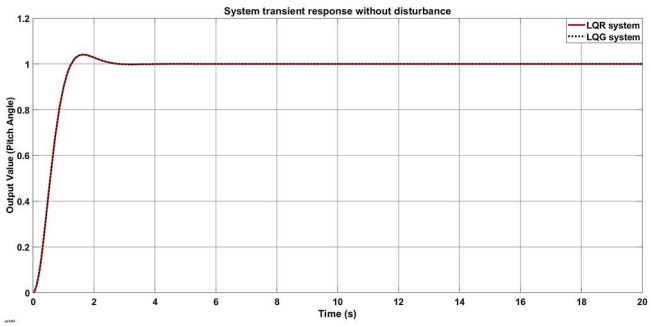

Fig. 7. Closed-loop step response under nominal (noise-free) conditions.

Under ideal noise-free conditions (Fig. 7), both the LQR and LQG control architectures yield identical response trajectories, reducing the rise time from 3.14 s to 0.79 s, shortening the settling time from 17.5 s to 2.16 s, and constraining overshoot to 4.1%. These results confirm that incorporating the Kalman filter into the closed-loop system preserves the optimal transient speed of the LQR regulator in the absence of disturbances [2], [4].

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B. Robustness Under Disturbances (Noisy Scenario)

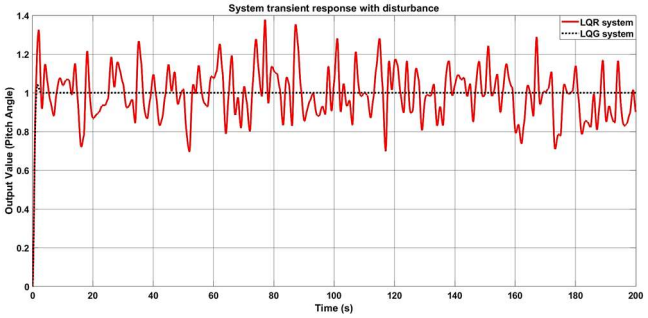

Fig. 8. Closed-loop step response under simultaneous process and measurement noise.

When simultaneous process and measurement noise are injected into the Simulink model (Fig. 6), a pronounced contrast in disturbance rejection capability emerges between the two architectures, as shown in Fig. 8:

LQR Control System: Because the directly fed-back state variables are heavily corrupted by noise, the control input experiences continuous high-amplitude fluctuations. Consequently, the output pitch angle oscillates severely between 0.70 rad and 1.38 rad, causing substantial tracking deviation from the 1 rad reference command.

LQG Control System: Leveraging the optimal stochastic filtering of the Kalman observer, random noise components are almost completely attenuated during state estimation prior to multiplication by the feedback gain K. Consequently, the pitch response maintains high stability, smoothly tracks the 1 rad reference command, and eliminates high-frequency chattering in the elevator actuator [1].

V. CONCLUSION

This study successfully designed and simulated an optimal LQG controller to stabilize the pitch attitude of the fixed-wing Ultra Stick 25E UAV operating under stochastic disturbances. Quantitative simulations in MATLAB/Simulink verify that integrating the LQR regulator with a Kalman filter not only preserves rapid transient dynamics (t_s = 2.16 s) and minimal overshoot (4.1%), but also provides superior noise filtering and disturbance rejection compared to conventional LQR control. These findings establish a reliable scientific foundation for further developing and experimentally validating robust autopilot control algorithms aboard lightweight UAVs operating in complex weather conditions.

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